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Analytical solution to the problem of heat transfer in an MHD flow inside a channel with prescribed sinusoidal wall heat flux

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Abstract

An MHD laminar flow through a two dimensional channel subjected to a uniform magnetic field and heated at the walls of the conduit over the whole length with a sinusoidal heat flux of vanishing mean value or not, is studied analytically. General expressions of the temperature distribution and of the local and mean Nusselt numbers are obtained by using the technique of linear operators in the case of negligible Joule and viscous dissipation and by taking into account the axial conduction effect. The principal results show that an increase of the local Nusselt number with Hartmann number is observed, and, far from the inlet section, the average heat transfer between the fluid and the walls shows a significant improvement at all values of Hartmann number used when the frequency of the prescribed sinusoidal wall heat flux is increasing in the case of vanishing mean value of the heat flux and this is true especially at low Peclet numbers.

Keywords: Laminar; MHD flow; Sinusoidal heat flux; Forced convection; Axial conduction; Linear operators; Analytical solution

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Nomenclature

b	half width between parallel plates channel (m)
$\vec{B}_0$	external uniform magnetic field vector (T)
k	thermal conductivity ($\mathrm{W\,m^{-1}\,K^{-1}}$)
M	Hartmann number, $B_0 b\sqrt{\frac{\sigma}{\mu}}$
Nu	local Nusselt number
$Nu_{\mathrm{as}}, \overline{Nu_{\mathrm{as}}}$	asymptotic Nusselt number and its mean value
p	pressure
Pe	Peclet number, $\frac{3U_{\mathrm{m}}b}{2\alpha}$
$q_{\mathrm{w}}(x)$	wall heat flux per unit area, $k\frac{\partial T}{\partial y}(x,b)$ ($\mathrm{W\,m^{-2}}$)
q_0	amplitude of wall heat flux ($\mathrm{W\,m^{-2}}$)
$T(x,y)$	temperature field (K)
T_0	uniform inlet section temperature (K)
$u(\eta)$	dimensionless velocity profile, $\frac{2}{3}\frac{u_x(y)}{U_{\mathrm{m}}}$
U_{m}	mean fluid velocity ($\mathrm{m\,s^{-1}}$)
$u_x(y)$	Hartmann velocity profile in axial direction, $U_{\mathrm{m}}\frac{\left(\mathrm{ch}\,M-\mathrm{ch}\,M\frac{y}{b}\right)}{\left(\mathrm{ch}\,M-\frac{\mathrm{sh}\,M}{M}\right)}$ ($\mathrm{m\,s^{-1}}$)
x	axial coordinate (m)
y	transverse coordinate (m)

Greek symbols

α	thermal diffusivity ($\mathrm{m^2\,s^{-1}}$)
β	frequency of wall heat flux ($\mathrm{m^{-1}}$)
δ	dimensionless parameter
γ	dimensionless parameter
η	dimensionless transverse coordinate, $\frac{y}{b}$
μ	dynamic viscosity ($\mathrm{kg\,m^{-1}\,s^{-1}}$)
θ	dimensionless temperature distribution, $\frac{T-T_0}{\frac{bq_0}{k}}$
θ_{b}	dimensionless bulk temperature
θ_{w}	dimensionless wall temperature
σ	electric conductivity ($\Omega^{-1}\,\mathrm{m^{-1}}$)
ω	dimensionless frequency, $\beta b Pe$
ξ	dimensionless axial coordinate, $\frac{x}{bPe}$

1. Introduction

Because of its great interest in the design of practical thermal systems, the problem of laminar forced convective flow heat transfer using a large number of fluids in a circular tube or parallel plates channel is of foremost importance. As is well known, the magnetic field influences the heat transfer and, thus, the temperature distribution through the change of the velocity of the fluid or liquid metal with the value of the Hartmann number M [1], which represents the ratio of the electromagnetic forces to the viscous forces. Periodic wall heat fluxes used in engineering applications