Available online at www.sciencedirect.com
SCIENCE DIRECT
European Journal of Mechanics A/Solids 24 (2005) 1030–1053

Development and experimental validation of a continuum micromechanics model for the elasticity of wood

Karin Hofstetter, Christian Hellmich *, Josef Eberhardsteiner

Institute for Mechanics of Materials and Structures, Vienna University of Technology (TU Wien), A-1040 Wien (Vienna), Austria

Received 29 November 2004; accepted 31 May 2005
Available online 1 September 2005

Abstract

This contribution covers the development and validation of a microelastic model for wood, based on a four-step homogenization scheme. At a length scale of several tens of nanometers, hemicellulose, lignin, and water are intimately mixed, and build up a polymer (polycrystal-type) network. At a length scale of around one micron, fiberlike aggregates of crystalline and amorphous cellulose are embedded in an contiguous polymer matrix, constituting the so-called cell wall material. At a length scale of about one hundred microns, the material softwood is defined, comprising cylindrical pores (lumen) in the cell wall material of the preceding homogenization step. Finally, at a length scale of several millimeters, hardwood comprises larger cylindrical pores (vessels) embedded in the softwood-type material homogenized before. Model validation rests on statistically and physically independent experiments: The macroscopic stiffness values (of hardwood or softwood) predicted by the micromechanical model on the basis of tissue-independent ('universal') phase stiffness properties of hemicellulose, amorphous cellulose, crystalline cellulose, lignin, and water (experimental set I) for tissue-specific composition data (experimental set IIb) are compared to corresponding experimentally determined tissue-specific stiffness values (experimental set IIa).

Keywords: Wood; Hierarchical organization; Chemical composition; Stiffness; Continuum micromechanics; Experimental validation

1. Introduction

Macroscopic mechanical properties of wood are characterized by a wide variability and diversity (Bodig and Jayne, 1982; Kollmann and Côté, 1968; Dinwoodie, 1981). Since this variability results from differences observed at the macro-, micro-, and ultrastructural scale, it has been striven for long to relate the macroscopic mechanical behavior to physical quantities at lower scales. The most common approach is that of cellular solids (Gibson and Ashby, 1997), relating Young's moduli to apparent densities of wood. While elucidating impressively certain load carrying characteristics of (micro-)beam compounds, errors of more than 1000% (see Gibson and Ashby, 1997, p. 403, Fig. 10.12) between experimental values and theoretical estimates render this theory as a qualitative rather than a precise quantitative tool.

Therefore, some authors invested into more detailed descriptions of the microstructure of wood, based on laminate theory for the representation of the internal structure of the cell wall (Yamamoto et al., 2002; Bergander and Salmén, 2000; Harrington et

* Corresponding author. Tel.: +43 1 58801 20220; fax: +43 1 58801 20299.
E-mail addresses: karin.hofstetter@tuwien.ac.at (K. Hofstetter), christian.hellmich@tuwien.ac.at (C. Hellmich), josef.eberhardsteiner@tuwien.ac.at (J. Eberhardsteiner).

0997-7538/$ – see front matter
doi:10.1016/j.euromechsol.2005.05.006

al., 1998). However, many of the required material data are not known from experiments. Hence, material parameters have to be guessed (e.g. Bergander and Salmén, 2002, Tables I and II; Yamamoto et al., 2002, Tables 1 and 2; Harrington et al., 1998, Table 2), which limits the applicability of such models; in particular, it does not permit model predictions for non-tested conditions.

In the present work, we aim at contributing to more reliable model predictions, decreasing the high prediction errors encountered with cellular solids theory. For this purpose, we re-visit the entire hierarchical organization of clear (knot-free) wood (rather than focusing on just the level of cell compounds as done in the cellular solids approach or on the level of the composite cell wall as done using laminate theory), in order to propose a first continuum micromechanics model for (clear) wood elasticity in the linear range. We further restrict our investigations to samples with empty lumen and vessel pores. Water is only considered within the cell wall, i.e. at an observation scale below one micron. Inelastic or time-dependent material behavior is beyond the scope of the present work.

The success of similar micromechanical models for bone (Hellmich and Ulm, 2002; Hellmich et al., 2004a, 2004b; Hellmich, in press);and the often stressed similarities between wood and bone (Fratzl, 2003; Ashby et al., 1995) seem to support our research endeavors, which we describe in the following.

2. Fundamentals of continuum micromechanics

In continuum micromechanics (Zaoui, 2002; Suquet, 1997), a material is understood as a micro-heterogeneous body filling a representative volume element (RVE) with characteristic length l, $l \gg d$, d standing for the characteristic length of inhomogeneities within the RVE (see Fig. 1). The 'homogenized' mechanical behavior of the material, i.e. the relation between homogeneous deformations acting on the boundary of the RVE and resulting (average) stresses, can then be estimated from the mechanical behavior of different homogeneous phases (representing the inhomogeneities within the RVE), their dosages within the RVE, their characteristic shapes, and their interactions. Based on matrix-inclusion problems (Eshelby, 1957; Laws, 1977), an estimate for the 'homogenized' stiffness of a material reads as (Zaoui, 2002)

$$\mathbb{C}^{\text{est}} = \sum_r f_r \mathbb{c}_r : \left[\mathbb{I} + \mathbb{P}_r^0 : \left(\mathbb{c}_r - \mathbb{C}^0\right)\right]^{-1} : \left\{\sum_s f_s \left[\mathbb{I} + \mathbb{P}_s^0 : \left(\mathbb{c}_s - \mathbb{C}^0\right)\right]^{-1}\right\}^{-1}, \tag{1}$$

where $\mathbb{c}_r$ and f_r denote the elastic stiffness and the volume fraction of phase r, respectively, and $\mathbb{I}$ is the fourth-order unity tensor. The two sums are taken over all phases of the heterogeneous material in the RVE. The fourth-order tensor $\mathbb{P}_r^0$ accounts for the characteristic shape of phase r in a matrix with stiffness $\mathbb{C}^0$. Choice of this stiffness describes the interactions between the phases: For $\mathbb{C}^0$ coinciding with one of the phase stiffnesses (Mori–Tanaka scheme), a composite material is represented (contiguous matrix with inclusions); for $\mathbb{C}^0 = \mathbb{C}^{\text{est}}$ (self-consistent scheme), a dispersed arrangement of the phases is considered (typical for polycrystals). If a single phase exhibits a heterogeneous microstructure itself, its mechanical behavior can be estimated by introduction of RVEs within this phase, with dimensions $l_2 \ll d$, comprising again smaller phases with characteristic length $d_2 \ll l_2$, and so on (see Fig. 1). This leads to a multistep homogenization scheme. Such a procedure should, in the end, provide access to 'universal' phase properties inherent to all wood tissues, at a sufficiently low observation scale. In order to identify appropriate RVEs and phases within wood, we will shortly recall its hierarchical organization.

3. Hierarchical organization of wood

Following pertinent work in the field (Fengel and Wegener, 2003; Kollmann and Côté, 1968; Wagenführ and Schreiber, 1974; Niklas, 1992), one may distinguish five levels of organization:

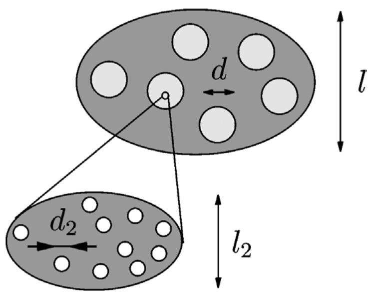

Fig. 1. Multistep homogenization.