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Material instabilities of anisotropic saturated multiphase porous media

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Abstract

This paper analyses the material instability of fully saturated multiphase porous media. On account of the fact that anisotropic mechanical behaviours are widely observed in saturated and partially saturated geomaterials, the anisotropic constitutive model developed by Rudnicki for geomaterials is used to model the anisotropic mechanical behaviour of the solid skeleton of saturated porous geomaterials in axisymmetric compression test. The inertial coupling effect between solid skeleton and pore fluid is also taken into account in dynamic cases. Conditions for static instability (strain localisation) and dynamic instability (stationary discontinuity and flutter instability) of fully saturated porous media are derived. The critical modulus, shear band angle for strain localisation, and the bound within which flutter instability may occur are given in explicit forms. The effects of material parameters on material instability are investigated in detail by numerical computations.

Keywords: Porous medium; Anisotropy; Strain localisation; Stationary discontinuity; Flutter instability

1. Introduction

The phenomena of material instabilities have been investigated by many researchers since the last century. The pioneering works for single-phase solids can be found in the papers by Hadamard (1903), Hill (1962) and Thomas (1961). Rice (1975, 1976) and Rudnicki (1983) investigated the instability of flow of saturated inelastic porous media in idealised initial and boundary value problems in quasi-static states. The dynamic contexts can be found in the works by Vardoulakis (1986, 1996), Schrefler et al. (1996), Zhang et al. (1999, 2001) and Zhang and Schrefler (2000, 2001, 2004). With a different approach, namely by analysing waves propagation in porous media, Loret and Harireche (1991) analysed the nature of the wave-speeds and the propagation modes of the acceleration waves in elastic-plastic fluid saturated porous media. It was shown that there are two modes in which the dynamic equilibrium equations loose their hyperbolic character; these modes are called stationary discontinuity (one wave is zero) and flutter instability (the squares of two wave speeds are complex conjugate). The former corresponds to the strain localisation condition characterising material instability in static or quasi-static cases. The growth and decay of the acceleration waves were studied by Loret et al. (1997). Simoes et al. (1999) established the relationship between the results of

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the analysis based on harmonic waves and those based on acceleration waves; further the reasons for the growth of acceleration waves in the non-associative case were investigated. Benallal and Comi (2002) used a linear perturbation approach to study the material instabilities in inelastic saturated porous media under dynamic loading. Loret and Rizzi (1998) analysed the effects of inertial coupling on the wave speeds of saturated elasto-plastic media. Li et al. (2002) investigated the instability of wave propagation in saturated elastic-plastic porous media in plane strain case by employing Drucker–Prager constitutive model.

On the other hand, anisotropic mechanical behaviour is widely observed in geomaterials owing to their particular fabric elements such as bedding, layering, foliation and lamination planes, or the existence of linear structures. However, so far the investigations of material instability are mostly restricted to isotropic elastic-plastic porous media, Few work considered anisotropy effect on the strain localisation and flutter instability. Rudnicki (1977) presented an anisotropic constitutive relation and investigated the effect of stress-induced anisotropy on the localisation of deformation of brittle rocks. Hutchinson and Miles (1974) used a similar relationship to investigate necking bifurcation in elastic-plastic cylinders subjected to uniaxial tension. Although the constitutive relation was used to interpret the behaviour of brittle rocks, it could be applied also to other materials such as metals, composites and polymers. Several researchers have used this relation to investigate bifurcation analysis including geometric diffuse modes and localisation of deformation. Miles and Nuwayhid (1985) used the same relation in their research. Chau (1992, 1993) used this relation to investigate the symmetric and antisymmetric bifurcations in a compressible pressure-sensitive circular cylinder under axisymmetric tension and compression, where a cylindrical polar coordinate (r, θ, z) was used to replace the Cartesian form. In a recent work, Rudnicki (2002) re-examined this relation for the possibility of compaction bands and shear bands in transversal isotropic solid materials. On the other hand, Rizzi and Loret (1999) found that the onset of strain localisation in anisotropic elastic-plastic porous media saturated by one fluid is coincident with that of the underlying drained solid, whatever the degree and the form of anisotropy of the solid skeleton. Bigoni and Loret (1999) and Bigoni et al. (2000) analysed successfully the influence of elastic anisotropy on the elastic-plastic acceleration wave speeds in plastic solids when the elastic anisotropy is described by a second order fabric tensor.

This paper extended the constitutive relation and the methodology proposed by Rudnicki (2002) (see also Miles and Nuwayhid, 1985 and Chau, 1993), where a transversely isotropic constitutive relation is used to investigate the conditions for the onset of shear bands in the axisymmetric compression test of single phase rocks, to model the material instability and strain localisation behaviours of saturated multiphase geomaterials. Bifurcation conditions used in the following result from the work by Vardoulakis (1983), Miles and Nuwayhid (1985), Chau (1992, 1993) and Rudnicki (2002) for axisymmetric loading cases, even if the constitutive law is nonclassical. Particular attention is focus on the effect of the liquid phase on the onset of strain localisation of multiphase materials. The explicit critical modulus, critical orientation to trigger strain localisation and the bounds in which flutter instability may occur are respectively investigated.

2. Governing equations for fully saturated multiphase porous media

In the framework of Biot's theory and with the assumption of isothermal behaviour, the governing equations of fully saturated porous media may be written as (Lewis and Schrefler, 1998)

$$\begin{aligned}
&\sigma''_{ij,j} - \bar{\alpha} p_{,i} - \rho \ddot{u}_i - \rho^w (\ddot{U}_i - \ddot{u}_i) = 0, \\
&-n p_{,i} - \rho^w \ddot{U}_i = n^2 \gamma_w k^{-1} (\dot{U}_i - \dot{u}_i), \\
&\frac{\dot{p}}{Q} + (\bar{\alpha} - n)\dot{u}_{i,i} + n\dot{U}_{i,i} = 0,
\end{aligned} \tag{1}$$

where n is the porosity, $\bar{\alpha}$ is the Biot parameter, $\rho = n\rho_w + (1-n)\rho_s$, $\rho^w = n\rho_w$, ρ_s, ρ_w are the densities of solid and fluid respectively, k the permeability coefficient and the parameter $Q = [n/K_w + (\bar{\alpha} - n)/K_s]$, with K_w, K_s the bulk modulus of pore water and solid grains respectively, p_w is pore water pressure, σ''_{ij} are the components of generalised effective stress, and γ_w the bulk density of pore water. U_i, u_i are real displacements of pore water and solid skeleton. The first equation of (1) is the linear momentum balance equation for multiphase media, the second one the linear momentum balance equation for pore water (Darcy's equation) and the third one the mass balance equation for water. The simplified equations neglect the effects of gravity and convective terms.

3. The anisotropic constitutive relation for the solid skeleton

Consider an axisymmetric compression test (see Fig. 1); the specimen is assumed to be oriented such that the x_3 direction is the axis of symmetry. The constitutive relation used by Rudnicki (1977) is given by the following relation between the components of the rate of deformation tensor d_{ij} and the components of Jaumann rate of Cauchy stress σ''_{ij}