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Limit analysis and Gurson's model

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Abstract

The yield criterion of a porous material using Gurson's model [Gurson, A.L., 1977. Continuum theory of ductile rupture by void nucleation and growth – Part I: Yield criteria and flow rules for porous ductile media. ASME J. Engrg. Mater. Technol. 99, 2–15] is investigated herein. Both methods of Limit Analysis are applied using linear and conic programming codes for solving resulting non-linear optimization problems. First, the results obtained for a porous media with cylindrical cavities [Francescato, P., Pastor, J., Riveill-Reydet, B., 2004. Ductile failure of cylindrically porous materials. Part 1: Plane stress problem and experimental results. Eur. J. Mech. A Solids 23, 181–190; Pastor, J., Francescato, P., Trillat, M., Loute, E., Rousselier, G., 2004. Ductile failure of cylindrically porous materials. Part 2: Other cases of symmetry. Eur. J. Mech. A Solids 23, 191–201] are summarized, showing that the Gurson expression is too restrictive in this case. Then the hollow sphere problem is investigated, in the axisymmetrical and in the three-dimensional (3D) cases. A plane mesh of discontinuous triangular elements is used to model the hollow sphere as RVE in the axisymmetrical example. This first model does not provide a very precise yield criterion. Then a full 3D model is applied (using discontinuous tetrahedral elements), thus solving nearly exactly the general three-dimensional problem. Several examples of loadings are investigated in order to test the final criterion in a variety of situations. As a result, the Gurson approach is slightly improved and, for the first time, it is validated by our rigorous static and kinematic approaches.

Keywords: Porous media; Gurson's model; Limit analysis; Lower and upper approach; Linear and conic programming

1. Introduction

Concerning the ductile failure of porous materials, Gurson's criterion (Gurson, 1977) is the most widely accepted because it is based on a homogenization method and on the kinematic approach of limit analysis. The plastic domain is approached from the outside by a semi-analytical approach, proved to be an upper bound approach by Leblond (2003).

Gurson's model treats a hollow sphere with macroscopic strain imposed on the boundary. The criterion that he proposed for a rigid plastic isotropic matrix around a spherical cavity is expressed as follows:

$$\frac{\Sigma_{\mathrm{eqv}}^2}{3k^2} + 2f\cosh\left(\frac{\sqrt{3}\,\Sigma_m}{2k}\right) = 1 + f^2 \tag{1}$$

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where f is the porosity rate of the material studied and k the flow stress in shear or cohesion. Σ_{eqv} is the macroscopic equivalent stress and Σ_m the macroscopic mean stress. By definition, this model does not take into account interactions between cavities.

The original Gurson criterion for porous materials has undergone various modifications to improve its adequacy with experimental and numerical results. Tvergaard (1981) uses empirical parameters to take into account cavity growth and the cavities coalescence. The Richmond and Smelser (1985) criterion introduces a parameter that can take into account shear bands, which could dominate the plastification process. The Sun and Wang (1989) criterion is the only one to be based on an inner approach of the Gurson model under a macroscopic stress on the hollow sphere boundary. The model of Gologanu et al. (1997) extended the Gurson model to the spheroidal void incorporating void shape effects.

The Rousselier (2001) criterion is based on a macroscopic thermodynamic approach of nonreversible phenomena under the local equilibrium assumption, i.e. the state of a system can be defined locally using the same variables as at the equilibrium state. The parameter values of this criterion must be determined experimentally. Numerous models have been developed for metal processing, such as Kim and Kwon (1992). More recently, Bigoni and Piccolroaz (2003) has proposed a macroscopic yield function to agree with a variety of experimental data relative to soil, concrete, rock, metallic and composite powders, metallic foams, porous materials and polymers. Their yield function is presented as a generalization of several classical criteria, also giving an approximation of the Gurson criterion.

In Francescato et al. (2004) and Pastor et al. (2004), both approaches of limit analysis make it possible to determine the yield criteria of a porous material with cylindrical cavities. In this case of porous material, we have shown that a Gurson-like formulation cannot properly represent the general solution. A realistic formulation must be specified in terms of at least three loading parameters, the Gurson expression being too restrictive.

In the present paper, the same tools of homogenization, limit analysis and optimization are used to obtain the plastic domain in the case of a porous material with *spherical* cavities. The Gurson model idealizes the porous material as a single cavity in a homothetic cell composed of a rigid plastic Mises material, called the Representative Volume Element (RVE) in the following. The homogenization method links macroscopic stress and strain rates Σ_{ij} and E_{ij} with microscopic stress and strain rates σ_{ij} and v_{ij} by the averaging relations:

$$\Sigma_{ij} = \frac{1}{V}\int_V \sigma_{ij}\,\mathrm{d}V, \qquad E_{ij} = \frac{1}{V}\int_V v_{ij}\,\mathrm{d}V. \tag{2}$$

This RVE is submitted either to a an average strain rate on the boundary of RVE ($u_i = E_{ij}x_j$, uniform strain rate loading) or to an average stress ($\sigma_{ij}n_j = \Sigma_{ij}n_j$, uniform stress loading) leading to different mechanical problems. Here, we essentially analyse the first case, i.e. $u_i = E_{ij}x_j$ on the RVE boundary, for testing the original Gurson criterion. Then we use the limit analysis static (or lower bound), and kinematic (or upper bound), approaches. They lead to non-linear optimization problems (because of the plasticity criterion), second order conic programing problems (SOCP) in fact. They are solved either by XA (after a pre-linearization of the non-linearities) or directly by MOSEK (an SOCP code), both optimization codes based on so-called Interior Point methods. These optimizers are the result of intensive algorithmic research since the pioneering work of Karmarkar in 1984. XA is a linear programming code: a pre-linearization is done using the very efficient Ben-Tal and Nemirovski (2001) method detailed in Pastor et al. (2004). The drawback of this linearization is the amount of memory required for the resulting large-scale problems. On the contrary, MOSEK solves the problem directly and therefore can deal with thinner RVE mesh.

After recalling the main results for porous materials with cylindrical cavities in Section 2, the admissible loading domain of the Gurson problem is investigated using the hollow sphere as RVE. In Section 3, the problem is considered an axisymmetric one (subject to an axisymmetric loading), using a plane mesh (and triangular elements) to discretize the RVE. This first model does not provide a sufficiently accurate yield criterion. Therefore, in Section 4, a three-dimensional mesh (and tetrahedral elements) is used. These models give a quasi-exact yield criterion, improving the Gurson criterion slightly. They do not confirm the corner found in Francescato et al. (2004) and Pastor et al. (2004) on the average stress axis in a porous material with cylindrical cavities.

More precisely, the kinematic 3D result is very close to the kinematic axisymmetric bound, which slightly improves the Gurson approach. This proves that the kinematic axisymmetric approach is excellent. The static 3D approach improves the static axisymmetric approach and becomes very close to the kinematic approach. Then the Gurson criterion, for the first time, is validated as a good approximation with both our kinematic and static approaches. Moreover, the kinematic axisymmetric approach is reinforced as a good prediction tool of the macroscopic isotropic criterion.

Furthermore, we confirm that, at least in the case studied, Gurson's criterion does not depend on the third invariant of the stress tensor. Finally, using the 3D model, macroscopic plane strain and plane stress loadings are also investigated.