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Multi-mode propagation and diffusion in structures through finite elements

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Abstract

The present work addresses the question of guided wave properties in structures. Guided multi-modal dispersion curves and diffusion of waves at singularities are the main concern. A propagative approach, based on a finite element model, is formulated. It provides an effective way to calculate the dispersion curves of complex guided structures. Some properties of guided waves are deduced from the formulation. The question of wave diffusion at substructures coupling locations is also considered. A closed formulation of the problem using Lagrange multipliers is presented. Its numerical implementation is detailed. Ultimately, a numerical test is offered. The case study concerns a thin walled structure.

Keywords: Wave finite element; Diffusion matrix; Lagrange multipliers; Guided waves

1. Introduction

Guided waves are still a subject of intensive research as such structural forms occur in several engineering areas. Accordingly, much of the existing literature focuses on the study of guided wave properties and guided wave applications. For instance, nowadays, such phenomena concern the field of acoustics, earthquake studies and electromagnetics, among many others. Applications of guided waves appear then in seismic problems, nondestructive testing but also in some relatively new areas such as optical fibers. Shapes and frequencies of such waves are strongly connected to the media studied and applications. The context covered in this paper is the linear elastodynamic field, and mainly vibroacoustic applications. Specifically, guided wave propagation and diffusion in complex elastic structures is the main matter of discussion of the present paper.

Among the primary properties of guided waves to be given are: the dispersion curve and the mode shapes of waves. Dispersion curves give the velocity-frequency relationship for all the modes which may propagate in the studied structure. The guided wave mode shape gives the distribution of displacements in a section normal to the propagation axis. The definition of these primary properties is one of the major questions dealt with in literature on guided waves. In addition to the properties described, guided waves interaction at boundaries or at interfaces between different structures, has relevance to several topics. For instance, in the context of modern vibroacoustics, specifically, if energy predictive tools are considered (among them, the well known predictive *Statistical Energy Analysis* (SEA) (Lyon, 1975; Keane and Price, 1997; Fahy, 1975, 1994), thus the main predictive analysis requires wave characteristics to be known: energy transmission

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coefficients, energy velocities and modal densities are to be given (Finnveden, 2004). Various approaches to the problem of computing guided wave properties (dispersion curves and diffusion matrices) are available. However, most of them are rather physical and analytical and so are restricted to very simple geometries and to very classical waves (see for instance Ref. (Mace, 1984) concerning the diffusion properties of two coupled homogeneous beams and Ref. (Von Flottow, 1986; Yong and Lin, 1989) concerning the diffusion properties of two coupled periodic models). So, one of the main objectives of the present contribution is to offer a full numerical description of such properties through the uses of a finite element model. In this context, it should be stated that the question of dispersion curve computation using FEM has raised some interest in the literature, whilst the question of diffusion matrix definition has not. Hence the motivation of this paper.

In the existing literature, dispersion curve computation through finite elements is not something completely new. Indeed many authors have been interested in a numerical determination of dispersion curves for particular structures. Among those contributors, should be noted the work of Gavric (1994; 1995) and the contribution of Knothe et al. (1994). Gavric and Knothe applied this extraction technique for rail structures. In Gavric (1994, 1995), Gavric proposed a particular finite element scheme allowing the extraction of wavenumbers from the resolution of a fourth order matrix equation. In Knothe et al. (1994), Knothe, introduced a numerical scheme well suited to infinite rail structures. The use of the latter allows dispersion curve extraction from a well posed eigenproblem. However, both Gavric and Knothe run into some numerical difficulties. Indeed, the first technique requires the development of a relatively new finite element code with specific elements, interpolation forms and an adapted eigenvalue extraction method. As for the second method, the structures are designated by an infinite succession of identical hyperelements, so that the cross-sections of each side are the same and the resulting mesh has internal nodes. This leads to a complex computational problem. Some interesting work on the problem at hand can be found in the literature on periodic structures and systems. Among them is Lin's work (Lin and Donaldson, 1969). Lin proposed in Lin and Donaldson (1969) a procedure, using transfer matrix concept, in which beamlike structures as well as curved panel examples were examined. This method runs into difficulties due to the inversion of ill conditioned matrices and the cumulative errors due to the transfer matrix assembly. Mead (1973, 1975) introduced fundamental and central ideas in the area of periodic systems characterization. In this context, Mead (1973) proposed a quadratic and well posed spectral problem in order to determine wave propagation constants of a periodic system. This work was extended in Denke et al. (1975) and in Mead (1975). Denke et al. (1975) and Mead (1975) proposed a second order matrix equation leading to the propagation constants of a periodic system. Thompson (1993) used the same technique after adding a damping contribution. Finally, the interesting work by Zhong et al. (Zhong and Williams, 1995; Zhong and Zhong, 1991) should be mentioned. Zhong (Zhong and Williams, 1995) offered a new eigenvalue problem, which is a kind of state space eigenproblem, wherein the main parameters are the displacement at both sections of the considered system.

In this contribution a closed formulation using finite element modeling is offered for the numerical computation of both dispersion curves and diffusion matrices. The formulation is general and can be applied to any kind of structural guided waves of a homogeneous or periodic structure. An eigenvalue problem is then written to determine numerically the dispersion curves in the wavenumbers/frequencies domain. The properties of such an eigenvalue problem are first discussed. A major concern of dispersion curves and wave shapes versus frequency identification is discussed, and an original solution offered to draw such properties rigorously. The latter is also fundamental in defining and computing the diffusion matrix. Indeed, coupling conditions at waveguide interfaces are expressed in terms of Lagrange multipliers and included in the formulation. Correct distribution of wave mode vectors in incident and reflected parts allows the diffusion matrix to be expressed in a very general way. Some numerical experiments are then presented. A simple academic structure is chosen in view of validating the general formulation offered here. The chosen guide are 3D elastic bars which offer the possibility of some analytical developments and comparisons. Dispersion curves of both bars are first extracted and commented. Guided waves shapes are also shown. The computation of diffusion matrices with regard to the given waves is then performed. Two coupling cases are studied in view of comparison. In the first one the coupling is longitudinal, whilst the second considers flexural like elastic coupling. In both situations, the comparison, when feasible, shows very good agreements.

2. Wave propagation in a periodic waveguide through finite elements

This study is concerned with a description of the dynamical behavior of a slender structure, as illustrated in Fig. 1, which is composed, along a specific direction (say x-axis), of N identical substructures. Note that this general description can be applied to homogeneous systems whose cross-sections are constant. Each substructure is supposed to be elastic, linear and dissipative. The dynamics of the global system is formulated from the description of the waves propagating along the x-axis.

Let us consider a finite element model of a given substructure k ($k \in \{1, \ldots, N\}$) belonging to the waveguide (cf. Fig. 1). The left and right boundaries of the discretized substructure are assumed to contain n degrees of freedom (dof's). The kinematic variables, displacements $\mathbf{q}$ and forces $\mathbf{F}$, defined on these boundaries are denoted by $(\mathbf{q}_\mathsf{L}, \mathbf{q}_\mathsf{R})$ and $(\mathbf{F}_\mathsf{L}, \mathbf{F}_\mathsf{R})$, respectively, and are represented through state vectors $\mathbf{u}_\mathsf{L}^{(k)} = ((\mathbf{q}_\mathsf{L}^{(k)})^\mathsf{T}(-\mathbf{F}_\mathsf{L}^{(k)})^\mathsf{T})^\mathsf{T}$ and $\mathbf{u}_\mathsf{R}^{(k)} = ((\mathbf{q}_\mathsf{R}^{(k)})^\mathsf{T}(\mathbf{F}_\mathsf{R}^{(k)})^\mathsf{T})^\mathsf{T}$. It is supposed that the internal