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European Journal of Mechanics A/Solids 24 (2005) 644–660

Structured deformation of damaged continua with cohesive-frictional sliding rough fractures

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Received 7 November 2004; accepted 23 December 2004

Abstract

The influence of cohesive-frictional crack sliding on the response of quasi-brittle fractured bodies and, in particular, the shear-induced dilatation due to crack surmounting, are here considered by assuming an equivalent shape function for crack-lip roughness. The deformation is modeled at the continuum level *via* structured deformation theory, introducing relevant kinematical descriptors that are now set in the classical thermodynamic formulation for generalized standard materials. The resulting model lies in the "simplified model" class, since internal variables may be reduced to one scalar parameter associated with the smeared-crack slip. Remarkably, even in the presence of friction, the structured-deformation approach renders the model fully-associated in type, a property particularly relevant for F.E. implementation. In the simplest case no evolution of crack density and orientation is supposed, and despite this simplification, a good description of the response of cracked masonry or concrete walls under seismic-like shear are provided by the model, whose calibration is obtainable through *ad hoc* tests. Interesting instabilities in the shearing path are exhibited, suggesting as well a possible generalization of the Mohr–Coulomb criterion.

Keywords: Damage mechanics; Structured deformations; Rough cracks; Friction; Cohesion; Crack sliding; Quasi-brittle materials; Concrete; Masonry; Geomaterials

1. Introduction

Due to the effect of cohesive forces opposing fracture opening, a diffuse micro-crack framework – rather than a dominant main crack – is developed under load in materials like concrete, ceramics, geomaterials or masonry, hence the name "quasi-brittle" (Bazant and Planas, 1998) by which such materials are usually referred to. Substantial progress in the description of this degradation process has been made during the last 20 years (Jefferson, 2003) using damage mechanics theory (Lemaitre and Chaboche, 1984), which presents some advantages with respect to classical fracture mechanics.

Damage mechanics historically begun with pure "macro" approaches, in which some scalar parameter was used to describe the fall of elastic properties due to microcrack evolution (Kachanov, 1986). The main idea of the successive micro–macro

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0997-7538/$ – see front matter
doi:10.1016/j.euromechsol.2004.12.005

approach consisted in homogenizing some local fracture mechanics solutions at the representative volume element (RVE) scale. The actual crack distribution, affecting the macro-response (Kaufmann and Marty, 1998; Belletti, Cerioni and Iori, 2001), could be determined from the macro-stress or -strain state (Steigman, 1991; Mazars, 1986; Ragueneau, La Borderie and Mazars, 2000), or could be supposed established by the first overcoming tensile stress in the virgin material (Pang and Hsu, 1996). The homogenization may (or may not) take into account interference between cracks (Benveniste, Dvorak and Wung, 1989; Fond, Fléjou and Berthaud, 1995) in case of high crack density, while the evolution laws can (or cannot) suppose fixed or not the crack orientation, considering as internal variables crack opening and crack sliding in the irreversible process thermodynamic approach (Andrieux, Bamberger and Marigo, 1986; Halm and Dragon, 1996, 1998; Pensée and Kondo, 2001). Possible cohesive forces (due to fiber bridging or grain bridging, aggregate frictional interlock, crack overlap etc.) can as well be taken into account in the Barenblatt (1962) sense.

The above class of models usually assume that cracks are flat or penny shaped (Onat, 1984). Nevertheless, it is well known that in quasi-brittle materials cracks exhibit considerable roughness, usually due to small-size heterogeneities (such as sand or stone aggregates in concrete). What is more, the measured roughness presents well-known self-similarity properties (Maji and Wang, 1992; Schmittbuhl, Roux and Berthaud, 1994; Amitrano and Schmittbuhl, 2002), emphasizing once more that roughness should not be neglected in any damage model for quasi-brittle materials. Indeed, at the macro level many classical tests, such as the shearing of wall panels, exhibit shear-induced dilatations that are due to rough-crack opening due to lip-sliding. In the fracture mechanics field, relevant models (Patton, 1996; Dyskin and Galybin, 2001) have been proposed to reproduce this effect but, to our knowledge, no micro–macro damage model takes into account roughness at the micro-level in a consistent thermodynamic approach.

In general, the continuum description is not trouble-free. As a rule, the cohesive forces bridging the crack surfaces gradually diminish the wider the crack opening is, so that the resulting equivalent continuum exhibit a strain softening response, which may lead to some theoretical difficulties such as strain localization. A consistent way to perform the limit of discontinuous deformations, as the average distance between opening/sliding cracks goes to zero, is furnished by structured deformation theory, recently advanced by Del Piero and Owen (1993, 2000) in order to develop earlier ideas in a rigorous mathematical framework. Roughly speaking, a structured deformation is the combination of a classical "regular" deformation with a "singular" deformation, produced by micro-disarrangements. This distinction is important in view of a thermodynamically-motivated theory because crack sliding, and the consequent induced crack-opening, lead to plasticity-like deformation splitting, while preserving their original mechanical significance.

The model here proposed, albeit tentatively, represents a first attempt at respond to the demand of a damage model involving rough fractures. The proposed approach is based upon structured deformation theory and it is built within the irreversible process framework, following the generalized standard-material theory (Halphen and Nguyen, 1975). The model structure, besides assuring easy numerical implementation, allows a straightforward extension to contemplate other approaches in the field of damage models. For example, in this introductory paper the crack pattern is supposed to be assigned and invariable during the loading process, but an evolution law for cracks could be easily added without major modifications, and it will be done in further work.

The plan of the paper is as follows. Relying upon structured deformation theory, a simple mesoscopic model for cracked materials is presented in Section 2, motivated by micromechanical considerations on rough crack sliding. In Section 3, the structured-deformation description is interpreted in a thermodynamic framework for irreversible processes by considering crack sliding as an internal variable associated with a proper thermodynamic force, through which macroscopic dissipation and yield criteria are deduced. In Section 4 the theory is applied to cracked materials under shear and confinement, showing a surprisingly wide range of material instabilities. Finally, a possible way to calibrate the model according to the experimental evidence is discussed in the concluding section.

2. Micromechanics via structured deformation theory

For the sake of simplicity, this study will treat a two-dimensional case only. Consider a membrane element made of a quasi-brittle material in generalized plane stress, completely damaged by diffuse micro-cracks. Suppose that in any region $\Re$, whose diameter is much larger than the panel thickness and crack spacing, but much smaller than the panel size, the orientation, spacing and characteristic length of the micro-cracks is uniform. Fig. 2.1(a) represents, for example, a damaged concrete panel with the aforementioned properties. We surmise that under moderate loading neither new cracks are nucleated nor crack length varies, although crack gliding is allowed. The punctual stress and strain will be very complicated, but an average view of such complex phenomena can be obtained by considering an *equivalent* crack layout for the panel, where cracks are parallel, equidistant and with the same profile. A representative volume element RVE for such an *ideal* material is drawn in Fig. 2.1(b). The orthogonal base $\{\mathbf{e}_1, \mathbf{e}_2\}$, used in the following, has been chosen such that $\mathbf{e}_1$ is at right angle to the crack average plane.

Let $\omega:\mathbb{R} \to \mathbb{R}$ represent a continuously differentiable, symmetric, even, periodic function, with mathematical period p. With respect to a (ξ, η) reference system, with ξ and η parallel to $\mathbf{e}_2$ and $\mathbf{e}_1$ respectively, the crack-lip shape is identified by