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The invariant manifold approach applied to nonlinear dynamics of a rotor-bearing system

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Abstract

The invariant manifold approach is used to explore the dynamics of a nonlinear rotor, by determining the nonlinear normal modes, constructing a reduced order model and evaluating its performance in the case of response to an initial condition. The procedure to determine the approximation of the invariant manifolds is discussed and a strategy to retain the speed dependent effects on the manifolds without solving the eigenvalue problem for each spin speed is presented. The performance of the reduced system is analysed in function of the spin speed.

Keywords: Nonlinear rotordynamics; Invariant manifolds; Reduced system; Normal modes

1. Introduction

The increasing need of optimized performance of structural systems enhance the importance of the nonlinear effects on their dynamics. To avoid the use of oversized components in obtaining optimal functioning of their systems, the designers and analysts began to extend the linear models to incorporate some kind of elements to reproduce the nonlinear peculiarities.

For linear structures the modal analysis technique is one of the most valuable tools and from its results the response of one structure may be found by solving ordinary differential equations with constant coefficients (Meirovitch, 1996). The key to this technique is to determine the linear transformation that takes the problem from one space where the description of the dynamical behavior is complex (in the sense where couplings between coordinates demand special care to solve the system of differential equations) to another space where the system of differential equations may be readily solved. The modal analysis allowed the development of reduction techniques that are very well developed nowadays. For rotor-bearing systems, perhaps the most simple and known reduction is the pseudo-modal method (Lalanne and Ferraris, 1990). This method of reduction retains the modal base of the system with null speed to describe the motions of the system at several spin speeds, and the reduction is achieved by retaining the low frequencies modes. Another kind of model reduction is the balanced model reduction (Mohiuddin et al., 1998).

The techniques mentioned above are well suited for linear systems. In rotor-bearing system there are many sources of nonlinearities, such as play in bearings, fluid dynamics in journal bearings, contacts between rotor and stator, among others

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Nomenclature

$\mathbf{C}$	damping matrix	a_{ij}, b_{ij}	coefficients for the approximation of the manifold
$\mathbf{G}$	gyroscopic matrix	c_{11}, c_{22}	elements of the damping matrix
$\mathbf{M}$	mass matrix	g_{12}, g_{21}	elements of the gyroscopic matrix
$\mathbf{K}$	stiffness matrix	m_{11}, m_{22}	elements of the mass matrix
$\boldsymbol{\Gamma}$	nonlinear coefficients matrix	k_{11}, k_{22}	elements of the stiffness matrix
$\mathbf{x}$	generalized coordinates	u	modal displacement
α	nonlinearity parameter	v	modal speed
Ω	spin speed		

(Yamamoto and Ishida, 2001; Ehrich, 1992; Vance, 1988). These phenomena lead to nonlinear differential equations of motion to express the dynamics of the system. Several methods are available, such as perturbation methods, harmonic balance methods, normal forms and center manifold methods (Nayfeh and Mook, 1979; Nayfeh and Balachandran, 1995; Guckenheimer and Holmes, 1986; Hsu, 1983a, 1983b; Szemplinska-Stupinicka, 1979, 1990; Yu, 1998; Jézéquel and Lamarque, 1991; Sinou et al., 2003a, 2003b, 2003c, 2004; Huseyin, 2002; Raghothama and Narayanan, 1999; Cameron and Griffin, 1989; Nelson and Nataraj, 1989). To try one of the reduction techniques, one approach is to linearize the system near to an equilibrium point. Clearly, this approach is valid when the vibrations are sufficiently small. In addition, it must be noted that the nonlinearities may lead to coordinate couplings that makes the linear modal reduction techniques difficult. Specialized methods for creating reduced models of nonlinear systems are available (Steindl and Troger, 2001), where the reduction is done with the aid of nonlinear Galerkin method and center manifold reduction.

The concept of nonlinear normal modes of vibration presents a great potential in system modeling and reduction techniques. Rosenberg's work (Rosenberg, 1966, 1962) is the cornerstone of the study of nonlinear normal modes. He defined the nonlinear normal mode for autonomous systems as one synchronous motion with fixed relations between generalized coordinates. In recent years, many studies explored the notion of nonlinear normal modes and nonlinear natural frequencies (Szemplinska-Stupinicka, 1979, 1990; Vakakis et al., 1996). The invariant manifold approach (Shaw and Pierre, 1991, 1993) brings the philosophy of the modal analysis to the nonlinear problems. In this approach a nonlinear normal mode is a motion that takes place on an invariant manifold that is tangent to the linear modal subspaces at the point of equilibrium. This definition leads to a nonlinear transformation that relates physical coordinates to nonlinear modal coordinates. The invariant manifold methodology has one aspect that seems to be very promising when searching for reduced models, since one nonlinear normal mode is constructed by projecting the other modes over it by means of a nonlinear relationship. These projections contain the nonlinear effects, and the performance of the reduced model is adapted to weak nonlinearities in virtue of the power series used to obtain the approximation. This approach allowed the incorporation of the nonlinear effects systematically into real world structures (Soares and Mazzilli, 2000).

This paper presents a numerical investigation of a nonlinear rotor-bearing system using the invariant manifold based methodology. As the solution of the eigenproblem of a rotor is dependent on the spin speed and the cost to search for it is very high when the dynamics of the system is to be known for several spin speeds, one strategy is presented to allow the determination of the linear invariant manifolds for several spin speeds based only in a few really calculated ones. By this strategy, a reduced model is constructed and its performance is evaluated and discussed.

2. The nonlinear normal modes

Consider a gyroscopic system with the following equation of motion:

$$\mathbf{M}\ddot{\mathbf{x}} + (\mathbf{C} + \Omega\mathbf{G})\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} + \boldsymbol{\Gamma}\mathbf{x}^{3^*} = \mathbf{0} \tag{1}$$

where $\mathbf{M}$ is the mass matrix (symmetric), $\mathbf{K}$ is the stiffness matrix (symmetric), $\mathbf{C}$ is the damping matrix (symmetric), $\mathbf{G}$ is the gyroscopic matrix (skew symmetric) and $\mathbf{x}$ is the vector of generalized coordinates. The term $\boldsymbol{\Gamma}\mathbf{x}^{3^*}$ represents a stiffness type nonlinearity, observing that 3^* means that the power acts over the elements of the vector and $\boldsymbol{\Gamma}$ is a diagonal matrix of coefficients. The matrices are given by

$$\begin{gathered}\mathbf{M} = \begin{bmatrix} m_{11} & 0 \\ 0 & m_{22} \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} k_{11} & 0 \\ 0 & k_{22} \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} c_{11} & 0 \\ 0 & c_{22} \end{bmatrix}, \\ \mathbf{G} = \begin{bmatrix} 0 & -g_{12} \\ g_{21} & 0 \end{bmatrix} \quad \text{and} \quad \boldsymbol{\Gamma} = \begin{bmatrix} \alpha & 0 \\ 0 & 0 \end{bmatrix}.\end{gathered} \tag{2}$$