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Optimal control of the rotational motion of a rigid body using Euler parameters with the help of rotor system

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Abstract

This paper presents a solution for optimal feedback regulation of a rigid body motion with the help of internal rotor system, taking into consideration the internal friction of the rotors. The optimal feedback control law is obtained as a function of the angular velocity and the orientation redundant parameters. Both of the angular velocity and the orientation parameters are regulated. In this study, stabilizer rotor system is used for regulation of the rigid body motion by the reaction rotors which can be developed by electric motors rigidly mounted on the body. Some known results on the optimal control of equilibrium positions of a rigid body are generalized and new results are obtained. Simulation results are presented.

Keywords: Optimal control; Rigid body; Redundant parameters; Rotors system; Internal friction

1. Introduction

The problem addressed in this paper has an important significance in aerospace science, since it corresponds to the control of the reorientation of a spacecraft. This problem received a great deal of attention in literature. The main thrust of previous research has been directed towards the time or fuel-optimal control problem; see for example Athans et al. (1963), Dixon et al. (1970), Branets et al. (1984), Junkins and Turner (1986). There are many early studies in this field that may be reported by Athans et al. (1963), Junkins and Turner (1986) and others.

Optimal control of a rigid body motion has a long history stemming mainly from the interest of aerospace engineers in the regulation of rigid spacecraft, satellite, space vehicle dynamics, robotic devices, gyroscopic instruments and many other applications of a rigid body.

Many different performance indices have been used to investigate the optimal regulation problem of a rigid body motion; see, for example, Krementulo (1966, 1977), Sakaain (1985), Tsiotras (1996), El-Gohary (2000a, 2000b, 2000c, 2000d, 2001).

Euler's parameters have many advantages over both Euler angles and direction cosines for determining the orientations of the rigid body (Stuelpnage, 1964; Morton et al., 1974). These parameters have no inherent geometrical singularity, there is no orientation of the body axes and inertial axes for which these parameters are undefined. Also, the elements of the direction cosine matrix are simple algebraic combinations of these parameters (Lur'e, 1961; Morton et al., 1974).

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It is well known that many relationships among physical quantities are not linear. One of the most important characteristics of nonlinear systems is the dependence of the system response behavior on the magnitude and type of the input. Nonlinear systems exhibit many phenomena that cannot be seen in linear systems, and in investigating such systems we must be familiar with these phenomena. Many different types of nonlinearities may be found in practical control systems, such as the motion of the rigid body, fluid motion and others (Ogata, 1970; Wittenburg, 1977).

The problems of optimal attitude regulation of a rigid body motion and system of rigid bodies are very important for numerous applications such as robotics, dynamics of satellite, spacecraft, aircraft, biomechanics and the like. Mortensen (1968), Krementulo (1977), Tsiotras (1996) have studied the problem of attitude of a rigid body rotation using different techniques. Also, El-Gohary (2000a, 2000b, 2000c, 2000d, 2001) has solved the problem of controlling the rigid body motion using different techniques. In these studies control laws are obtained by optimization; then the asymptotic stability is shown.

Known results on the regulation of the reorientation of a rigid body are generalized in this paper and new results are obtained. The main advantage of the stabilization schemes proposed here is their practical simplicity compared with previously proposed stabilization methods for the motion of controlled rigid body. Also the global asymptotic stability of the equilibrium position of a rigid body is proved.

In this paper the problem of optimal control of an equilibrium position of a rigid body with the help of internal rotor system is considered taking into account the internal noises friction moments arise from the dependence of coefficients of the rotors friction on various factors as temperature, angular velocity and others (Sagirow, 1970). In this study Euler redundant parameters are used for describing the orientations of the body. The optimal control moments, which ensure optimal asymptotic stability of this position are obtained as a function of phase coordinates of the body and coefficients of rotor friction. In this study, Liapunov technique is used to prove the global asymptotic stability of the desired equilibrium position.

2. Equations of motion

We consider rotational dynamics of a mechanical system $\mathcal{S}$ that consists of a rigid body $\mathcal{B}$ as a (platform) with a fixed point O as its center of mass and a system $\mathcal{R}$ of three symmetrical rotors which are attached to the principal axes of inertia of the system $\mathcal{S}$, in such a way that motion of the rotors does not modify the distribution of masses of the system $\mathcal{S}$. The rigid body controlled by the reaction rotors which can be developed by electric motors rigidly mounted on the body. To describe the motion of this system, we introduce two frames of references, the first $\{\hat{\mathbf{n}}\}$ is the inertial frame, and the second frame $\{\hat{\mathbf{b}}\}$ is moving with the body axes coincident with the principal axes of inertia of the system $\mathcal{S}$. The resulting rigid body dynamics are given by:

$$A_1\dot{\omega}_1 + (A_3\omega_3 + I_3\dot{\phi}_3)\omega_2 - (A_2\omega_2 + I_2\dot{\phi}_2)\omega_3 + I_1\ddot{\phi}_1 = M_1, \quad (123), \tag{2.1}$$

and the dynamics of the rotor system taking into account the internal friction moments of the rotors, with respect to the body frame $\{\hat{\mathbf{b}}\}$, are given by:

$$I_1(\dot{\omega}_1 + \ddot{\phi}_1) = L_1 - \nu_1\dot{\phi}_1, \quad (123), \tag{2.2}$$

where ω_i $(i = 1, 2, 3)$ are the components of the angular velocity vector $\vec{\omega}$ of the rigid body, expressed in the body-fixed frame $\{\hat{\mathbf{b}}\}$; $\dot{\phi}_i$ $(i = 1, 2, 3)$ are the relative angular velocities of the rotors with respect to their respective axes of rotation (the rigid body principal axes); L_i $(i = 1, 2, 3)$ are the control that applied to the rotors; M_i $(i = 1, 2, 3)$ are the components of the external moments vector $\vec{M}$ expressed in the body fixed frame $\{\hat{\mathbf{b}}\}$; A_i, I_i $(i = 1, 2, 3)$ are the principal moments of inertia of the complete system and axial moments of inertia relative to body frame $\{\hat{\mathbf{b}}\}$ respectively. Finally, the dot denotes the derivative with respect to time t and the notation (123) means that one of three equations is written and the others can be obtained from it by cyclic permutation of the indices $1 \to 2 \to 3 \to 1$.

In this study we assume that the external moments M_i are very small compared with the artificial internal control moments L_i (Sagirow, 1970) or there are no external moments affected on the body. Note that these moments usual ignored in the control design. The expressions $\nu_i\dot{\phi}_i$ $(i = 1, 2, 3)$ denote the frictional moments on the rotors. Note that the coefficients ν_i $(i = 1, 2, 3)$ depend upon various factors such as temperature, angular velocity and other factors. Thus, an exact model of the friction is very complicated.

The control moments L_i will be determined from the condition to ensure optimal stabilization of the desired equilibrium position of the rigid body. Our interest is in attitude control of the rigid body, so that Eqs. (2.2) for the reaction of rotors system will be used for elimination of the accelerations of the rotors from Eqs. (2.1).

In addition to the dynamics, which provides the time history of the angular velocity vector, orientation of the rigid body is given by the kinematic equations. Euler's Principal Rotation Theorem states that *a completely general angular displacement of a rigid body can be accomplished by a single rotation through an angle Φ about a unit vector $\hat{l}$ (principal line) which is*