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Evolution of complex singularities in Kida–Pelz and Taylor–Green inviscid flows

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Abstract

The analyticity strip method is used to trace complex singularities in direct numerical simulations of the Kida–Pelz and Taylor–Green flows, performed with up to 2048^3 collocation points. Oscillations found in the Kida–Pelz energy spectrum are attributed to interferences of complex singularities. A generalized least-square fit that separates out the oscillations from the measure of the width of the analyticity strip δ is introduced. Using the available resolution, δ is found to decay exponentially in time up to $t = 1.25$. It is argued that resolutions in the range 16384^3–32768^3 (within reach of the Earth Simulator) are needed to really probe the Pelz singularity at $t \sim 2$.

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1. Introduction

The existence of a finite-time infinite-vorticity singularity in three-dimensional incompressible Euler flow developing from smooth initial conditions is still an open mathematical problem (Frisch et al., 2003).

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One possible approach to this problem is the so-called analyticity strip method (Sulem et al., 1983). The basic idea of this method is to trace complex singularities numerically on direct numerical simulations (DNS) of the Euler equation with enough spatial resolution to capture the exponential tails in the Fourier transforms. The logarithmic decrement of the energy spectrum at high-k is twice the width $\delta(t)$ of the analyticity strip of the velocity field and the problem of blowup reduces to check if $\delta(t)$ vanishes in a finite time.

This method has been applied to three-dimensional Euler flows generated by the Taylor and Green (1937) (TG) initial conditions, with resolutions 256^3 (Brachet et al., 1983) and 864^3 (Brachet et al., 1992). It was observed that, after an early transient period, the width of the analyticity strip of the velocity field decayed exponentially in time.

The Kida–Pelz (KP) flow was introduced by Kida (1985). It has all the symmetries of the TG vortex and also displays additional symmetries that make it invariant under the full octahedral group (Pelz, 2001). This flow was used by Pelz (2003), Pelz and Gulak (1997a, b), Boratav and Pelz (1994b) to study the problem of Euler blowup, using temporal Taylor series expansions. It was also used by Boratav and Pelz (1994a) to make DNS of viscous turbulence.

It has been argued by Kerr (1993) that more symmetries than the ones present in the TG vortex are needed in order to observe a singularity. Thus, the KP flow could well be a better candidate for finite time singularity than the TG flow.

The main purpose of this paper is to apply the analyticity strip method to DNS of the TG and KP flows with resolutions up to 2048^3. It will turn out to be necessary to generalize the least square fit used to extract $\delta(t)$ from the energy spectrum so that the fit takes into account oscillations that are found in the KP energy spectrum.

The paper is organized as follows. Section 2 contains a short description of the (standard) numerical methods used to integrate the Euler equation. Section 3 contains the generalization of the least square fit and numerical results. Finally Section 4 is our conclusion.

2. Numerical approach

The three-dimensional incompressible Euler equations,

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla p, \tag{1}$$

$$\nabla \cdot \mathbf{v} = 0 \tag{2}$$

with (2π-periodic) initial data are solved numerically using standard (Gottlieb and Orszag, 1977) pseudo-spectral methods with resolution N. Time marching is done with a second-order leapfrog finite-difference scheme. The solutions are dealiased by suppressing, at each time step, the modes for which at least one wave-vector component exceeds two-thirds of the maximum wave-number $N/2$ (thus a 2048^3 run is truncated at $k_{\max} = 682$). Symmetries are used in a standard way (Brachet et al., 1983) to reduce memory storage and speed up computations.

Two types of computations, corresponding to different initial conditions, are carried out. The first type concerns the Taylor–Green vortex (Taylor and Green, 1937) which is the incompressible