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Excitation of polar thermal convection in a rotating spherical shell

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Abstract

The structure of thermal convection of a Boussinesq fluid with Prandtl number of 1 in a rotating spherical shell of inner-to-outer radius ratio of 0.5 is investigated numerically for the Rayleigh number Ra up to 90 times the critical value Ra_c at the Taylor number of 1.6×10^6. As long as Ra is close to Ra_c from above, a convective flow is driven only outside the polar regions. At $Ra \simeq 5Ra_c$, the convection starts also inside the polar regions where high temperature blobs are created, that enhance the heat transport. The convection in the polar regions is more active than that in the other region for $Ra > 10Ra_c$. The relative strength of convective motions in the two regions attains the largest value at $Ra \approx 20Ra_c$, beyond which the convection field becomes more isotropic as Ra increases.

Keywords: Thermal convection; Rotation; Spherical shell

1. Introduction

The thermal convection in a rotating spherical shell or a rotating sphere with a spherically symmetric gravity force has been studied extensively as a simple model of geodynamo (see Busse, 2000; Roberts and Glatzmaier, 2000; Zhang and Schubert, 2000; Busse et al., 2003). The outer core of the Earth is composed of liquid metals, the flow of which is believed to cause generation and reversals of the geomagnetic field. However, the dynamical relationship between the fluid motion and these geomagnetic phenomena has

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not been well understood yet because high Taylor number and high Rayleigh number make the flow very complicated in both space and time. The convection field exhibits diverse behaviour depending upon the system parameters, the fluid properties, and so on.

Over the years, a great deal of investigations have been conducted on the onset of convection both theoretically and numerically to reveal many characteristics of rotating convection. It was shown theoretically that the critical Rayleigh number above which a convection occurs increases as $Ta^{1/6}$ (*Ta* being the Taylor number) and the critical longitudinal mode number of convection is proportional to $Ta^{2/3}$ (Roberts, 1968; Busse, 1970; Dormy et al., 2004). In numerical approach, on the other hand, there are quite a few results for the full nonlinear convection system. Tilgner and Busse (1997) investigated the effects of the Prandtl number on finite-amplitude convection in a rotating spherical shell. They observed a substantial difference in the dependence of the Nusselt number and the kinetic energy on the Rayleigh number for different Prandtl numbers. Kitauchi et al. (1997) analyzed very convoluted topological patterns with reference to stagnation points and closed streamlines in a steady state convection. Glatzmaier and Roberts (1997) investigated the role of the inner core, which had been believed to be negligible or not considered for simplicity before, in a geodynamo simulation. They found that there are two convective regions in the rotating spherical shell divided by a *tangent cylinder* which is a cylinder tangent to the inner sphere at the equatorial plane. A convection with upwelling and azimuthal thermal wind is observed inside the tangent cylinder, whereas the Taylor–Proudman convection columns outside. (See Fig. 2 for the tangent cylinder.) Christensen et al. (1999) performed numerical simulations of dynamo in a rotating electrically conducting spherical shell for a wide range of parameters. They reported, as a preliminary result, that convection is most vigorous inside the tangent cylinder for the Rayleigh number in excess of 20 times the critical in the case of the Taylor number of 4×10^8, the Prandtl number of 1 and the magnetic Prandtl number of 2.

The purpose of the present paper is to understand the physical properties of convection when the convective state is far from the onset, at a relatively high Taylor number such that the convection columns extend to the outer boundary along the rotation axis. There are four control parameters in this system; the inner-to-outer radius ratio η of the spherical boundaries, the Taylor number *Ta*, the Rayleigh number *Ra*, and the Prandtl number *Pr*. See Eqs. (1) for the definitions of these parameters. The Rayleigh number characterizes the onset of convection of the system. Here, we investigate a qualitative change of thermal convection depending on *Ra* up to 90 times the critical value, while the other parameters are fixed. The structures of the convection field are examined by using flow visualization.

This paper is organized as follows. The mathematical formulation is introduced briefly in Section 2. The numerical results are presented in Sections 3 and 4. In Section 3, a dependence of the Nusselt number on the Rayleigh number is described. In Section 4, the structure of convection field is analysed in detail. Section 5 is devoted to concluding remarks.

2. Mathematical formulation

We consider the motion of a Boussinesq fluid confined in a spherical shell of thickness d which is rotating at a common constant angular velocity $\boldsymbol{\Omega}$ (see Fig. 1). The radii of the spheres of the inner and outer boundaries are denoted by r_{i} and $r_{\mathrm{o}}(=r_{\mathrm{i}}+d)$, respectively. The temperature on the inner and outer spheres is kept uniform and constant at all the times, and the inner sphere is hotter than the outer by ΔT. The gravity force $\boldsymbol{g} = -\gamma \boldsymbol{r}$ acts on the fluid per unit mass in the direction of the system centre, where $\boldsymbol{r}$ is