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Derivation of the strain energy release rate G from first principles for the pressurized blister test

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Abstract

The strain energy release rate, G, is derived from first principles and is shown to be consistent with the exact numerical solution by Cotterell and Chen (Int. J. Fract. 86 (1997) 191). Comparison with the classical Rayleigh–Ritz energy method and other published methods shows that the gradual change in blister profile over the entire loading process must be considered for the correct G to be calculated.

Keywords: C. Fracture mechanics; D. Delamination; C. Blister test; D. Fracture

1. Introduction

The first pressurized blister test was devised by Dannenberg in 1961 for measuring adhesion of paint on wall [2]. When a critical uniform pressure is applied through a hole in the substrate, the thin film blisters and delaminates from the rigid substrate (Fig. 1). The adhesion energy can be deduced from an energy balance method that relates instantaneous applied pressure, blister height, and delamination radius. The method also allows the evaluation of residual membrane stress [3–5]. The method is now widely used in the investigation of thin film adhesion and reliability of encapsulated devices and films in the microelectronics industry (e.g. CVD diamond [6], polyimide [7], metal [8], elastomeric adhesive [9]), and mechanical performance and behavior of bio-membranes and natural films (e.g. lipid bilayer [10], plasma [11], water monolayer at a "Janus interface" [12].)

The rigorous linear elastic fracture mechanics model was first constructed by Williams et al. [13, 14] for a blistering plate under bending. Hinkley [15] built a fracture model for thin and flexible film and deduced the strain energy release rate to be $G = \Phi p w_0$, with Φ a dimensionless fracture parameter, p the applied pressure and w_0 the central blister height. His calculation was corrected by Gent and Kaang [16], who adopted Hencky's membrane solution [17] as the constitutive relation, $p(w_0)$. Based on linear elasticity and an average stress approximation, Williams derived the fracture parameter for the blister test and a number of other related loading configurations [18]. Jensen took the subject further by considering the stress intensity factors and the associated mode mixity [19]. Cotterell and Chen [1] solved the von Karman equation by use of Way's method [20] for a blistering film that spaned a wide range of thickness. They calculated the constitutive relation, and derived G from an energy balance. Wan and Lim [21] recently adopted the average stress approximation to derive analytical forms of $p(w_0)$ and G without considering the loading history, despite a significant deviation from Cotterell's exact numerical solution. In this paper, we will resolve the inconsistency by deriving G from first principles from an energy balance. The loading history will be included in the calculation. The classical Rayleigh–Ritz energy method [22] will also be used to deduce $p(w_0)$ and G for comparison with our new analytical model.

2. Theory

Before proceeding to the mechanics of thin film delamination from first principles, we re-derive the constitutive relation *without* delamination for an axisymmetric plate based on linear elasticity and average membrane stress approximation, as in our earlier paper

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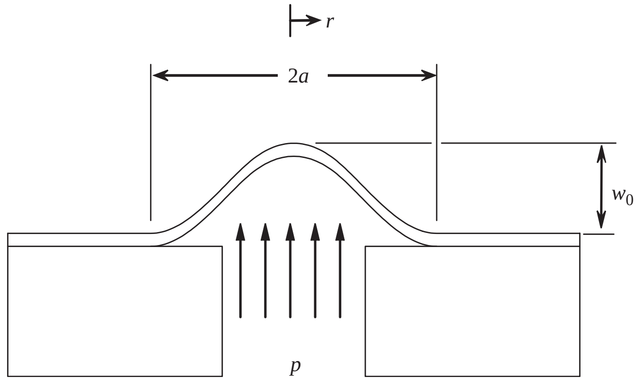

Fig. 1. Schematic drawing showing a thin blistering film delaminating from a rigid substrate under a uniform pressure, p.

[21]. Some typos in the previous derivation will be corrected and more details given.

2.1. Constitutive relation without delamination

A thin circular film of radius, a, area $A = \pi a^2$, and flexural rigidity $D = Eh^3/(1-\nu^2)$ with elastic modulus, E, Poisson's ratio, ν, and thickness, h, is fixed at the circumference with zero residual membrane stress (Fig. 1). A uniform pressure, p, is applied to deform the film by a bending moment, $-D\nabla^2 w$, and a membrane stress, σ, forming a blister with height, w_0, and volume, V. The constitutive relation can be derived by solving the von Karman equation to give either a series or numerical solution [23, 24]. An analytical solution based on an average stress approximation ($\sigma_r = \sigma_t = \sigma$) will be shown here. The blister profile as a function of the radial position, r, from the hole center, $w(r)$, was shown earlier to be governed by the linear elastic equation [21, 24, 25]

$$-D\nabla^4 w + (\sigma h)\nabla^2 w = p, \tag{1}$$

where ∇^2 is the Laplacian operator. A set of normalized quantities is conveniently defined as $\xi = r/a, \omega = w/h, \beta = (\sigma a^2/D)^{1/2}, \rho = pa^4/2Dh$, and $\upsilon = V/(\pi a^2 h)$. Integrating (1) with respect to r, in normalized quantities

$$\xi^2\omega''' + \xi\omega'' - (1+\beta^2\xi^2)\omega' = \rho\xi^3, \tag{2}$$

with the prime denoting $(\mathrm{d}/\mathrm{d}\xi)$. The standard modified Bessel equation (2) yields

$$\omega = \rho\left[\frac{I_0(\beta\xi) - I_0(\beta)}{\beta^3 I_1(\beta)} + \frac{1-\xi^2}{2\beta^2}\right], \tag{3}$$

with $I_n(x)$ being the nth-order modified Bessel equation of the first kind. The normalized blister height is

$$\omega_0 = \omega(\xi = 0) = \rho\left[\frac{1 - I_0(\beta)}{\beta^3 I_1(\beta)} + \frac{1}{2\beta^2}\right] \tag{4}$$

and the normalized blister volume is

$$\upsilon = \int_0^1 2\omega\xi\,\mathrm{d}\xi = \rho\left[\frac{2}{\beta^4} + \frac{1}{4\beta^2} - \frac{I_0(\beta)}{\beta^3 I_1(\beta)}\right]. \tag{5}$$

The constitutive relation is found by invoking the linear stress–strain relationship for a membrane,

$$\rho = \frac{2^{1/2}\beta^{11/2} I_1(\beta)}{\left[9\beta I_1(\beta)^2 - 6\beta I_0(\beta) I_1(\beta) - 24 I_1(\beta) I_2(\beta)\right]^{1/2}}. \tag{6}$$

Eliminating β from (4) and (6), it can be shown that

$$\rho = k(\beta)\upsilon^{n(\beta)}, \tag{7}$$

with $k(\beta)$ a proportionality constant and $n(\beta) = \mathrm{d}(\log\rho)/\mathrm{d}(\log\upsilon)$. An analytical form of $n(\beta)$ is available, but the lengthy expression is not explicitly given here. Figs. 2a–c show ρ,n and (υ/ω_0) as functions of ω_0, respectively. When the deformation is small ($\omega_0 \ll 1$),

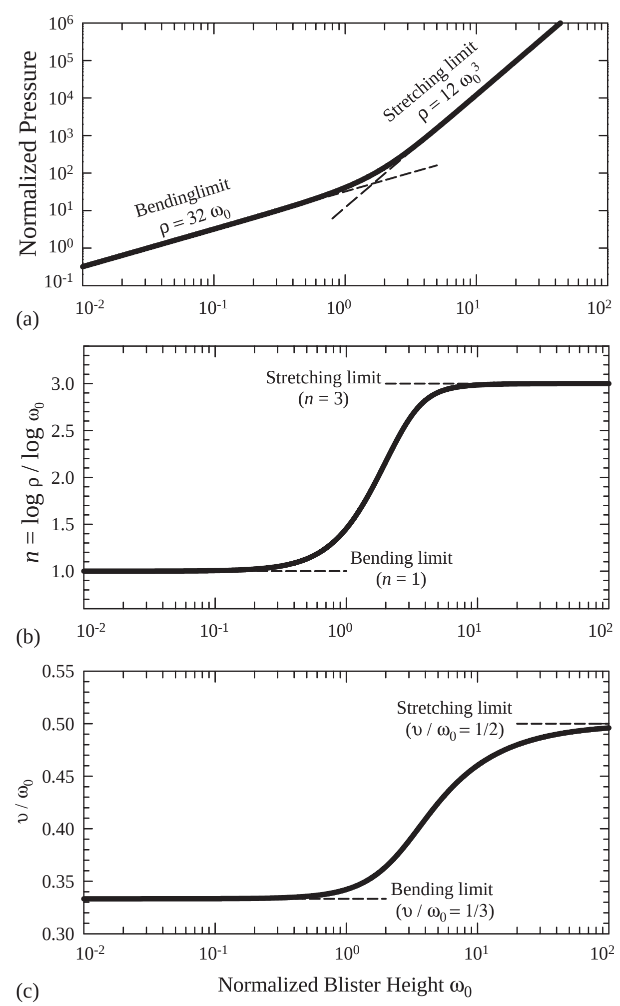

Fig. 2. (a) The constitutive relation $\rho(\omega_0)$ showing a linear behavior in the bending limit and a cubic behavior in the stretching limit. (b) Gradient of $\rho(\omega_0)$ as a function of ω_0. (c) The blister volume to height ratio as a function of ω_0. Bending-to-stretching transition of $\rho(\omega_0)$ and $n(\omega_0)$ occurs at $\omega_0 \approx 2$, while (υ/ω_0) makes the transition at $\omega_0 \approx 5$.