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Statistical distribution functions and fatigue of structures

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Abstract

Three statistical distribution functions are compared. The log(N) normal distribution, the 3-parameter Weibull distribution and the 3-parameter log($N-N_0$) normal distribution. The latter function is relatively new. Attention is paid to very low probabilities of failure. Various sources of scatter of fatigue test data and fatigue of structures in service are recapitulated. Different practical problems for which statistics are important are defined. Limitations of statistical predictions are briefly discussed. The significance of fatigue acceptance tests and realistic full scale service-simulation tests are emphasized.
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1. Introduction

The evaluation of statistical fatigue problems of structures and materials is addressed in numerous publications in the open literature. The variety of objectives includes essentially different purposes. The most-simple case is to characterize the variation of fatigue properties, such as the standard deviation of a fatigue limit. A practical case occurs in comparative test programs if the question is whether option A is statistically superior to option B. As an example, is the fatigue life of a joint with a new type of fasteners statistically superior to the life obtained with fasteners previously used? In the industry comparative fatigue tests are also carried out to check the quality of a material or a product procured from suppliers.

Another category of publications is dealing 'with designing against fatigue', predictions on fatigue, and safety issues. Statistical aspects must then be recognized. Safety factors to avoid fatigue failures of industrial products are to be used for reasons of economy and liability. This implies that very low probabilities of failure have to be considered. Sometimes the growth of fatigue cracks is part of a statistical scenario. Inspections should then prevent catastrophic failures.

In general, the statistical analysis of fatigue problems requires a statistical distribution function of the relevant variables. Unfortunately, statistical distributions for fatigue problems cannot be derived from a physical description of the fatigue phenomenon, in spite of the fact that the knowledge about fatigue damage of a material is fairly well developed in a qualitative way [1]. As a consequence, a statistical distribution function must be assumed. The most well-known function is the normal distribution (also Gaussian distribution). In this paper three distribution functions are considered; (i) the log(N)-normal distribution function, (ii) the 3-parameter Weibull distribution function, and (iii) the log($N-N_0$)-normal distribution function. The latter function appears to be a new one although it was proposed long ago. The three functions are used for data fitting to results of a two fatigue test series. Attention is paid to very low probabilities of failure. The paper continues with general observations about scatter of fatigue results caused by different sources. The significance of scatter for practical problems of industrial production and the structure in service is summarized and different types of planning fatigue tests are stipulated. The paper is completed with a number of conclusions.

2. Three statistical distribution functions

The most well known and classical distribution function is the normal distribution function, also called the Gaussian

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function. The equation for the probability of values $v \le x$ is given in Eq. (1).

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}\left(\frac{v-\mu}{\sigma}\right)^2} dv \tag{1}$$

In this equation μ is the mean value, σ is the standard deviation, and v the integration variable. A linear function between $P(x)$ and the variable x is obtained if this function is plotted on so-called normal probability paper, see Fig. 1. The slope of the straight line depends on σ. Usually, $\log(N)$ is adopted as the variable x for the evaluation of test results of similar fatigue tests.

Another popular distribution function is the 3-parameter Weibull distribution function [2], presented as Eq. (2).

$$P(x) = 1 - e^{-\left(\frac{x-x_0}{a}\right)^b} \tag{2}$$

There are three parameters in this equation. The function has a lower limit x_0 for which $P(x)=0$. It implies that fatigue lives smaller than N_0 (corresponding to $x_0=\log(N_0)$) cannot occur. This appears to be realistic from a physical point of view. The parameter a is called the scale parameter, and b the shape parameter in view of their effects on the probability density curve. In several publications x_0 is assumed to be zero and Eq. (2) then reduces to the 2-parameter Weibull distribution function. If a Weibull distribution is fitted to test results, three parameters obviously allow a better correlation with the data than two parameters. The 2-parameter Weibull distribution function will not be considered any further in this paper.

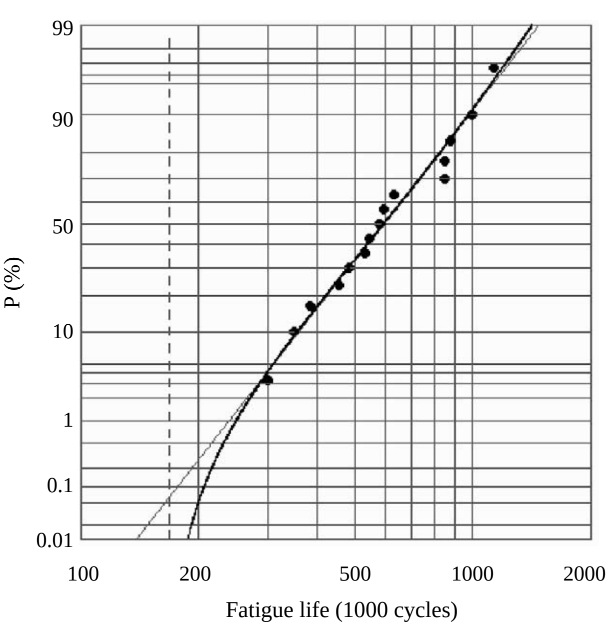

Fig. 1. Test results of 15 similar fatigue tests on welded specimens plotted in a normal probability graph [6]. Data fitting with the log(N)-normal distribution function (straight line) and the 3-parameter Weibull distribution function (non-linear curve, $N_0=168$ kc).

A second 3-parameter distribution function with a lower limit for the fatigue life will now be discussed. In his book Statistical Theory with Engineering Applications [3], Hald discussed skew distributions. According to Hald, distributions influenced by economics, psychological and biological factors are generally skew. Hald also states that theoretically it is always possible to determine a function $g(x)$ which will transform a skew distribution of x into a normal distribution. As a noticeable example, Hald discusses the logarithmic normal distribution with the transformation function $g(x)=\log(x)$. Actually, this applies to the $\log(N)$-normal distribution function discussed above. Hald also mentions the transformation function $g(x)=\log(x-a)$ with 'a' as an additional parameter of the logarithmic normal distribution. This function was already discussed in 1916 by Kapteyn and Van Uven [4]. They showed that statistical data of a skew distribution could be fitted by a linear relation between $P(x)$ and $\log(x-a)$ if plotted on normal probability paper. With the present notation of N for the fatigue life, the statistical variable x is:

$$x = \log(N - N_0) \tag{3}$$

The probability function is still the same as in Eq. (1) but now $x=\log(N-N_0)$, and μ and σ are the mean value and standard deviation of $\log(N-N_0)$, respectively. The distribution function is still a normal distribution, but now three parameters are involved. The attractive feature is that a lower limit of the fatigue life N_0 is introduced as a location parameter, although mathematically in a different way as in the Weibull 3-parameter equation. It may be appreciated that the normality of the distribution function is maintained.

The option of the $\log(N-N_0)$-normal distribution function apparently escaped the attention of papers on statistical evaluations of fatigue test results. De Jonge in 1983 [5] compared this distribution function with the 3-parameter Weibull distribution function. He showed that approximately similar shapes of the probability density function, depending on the 3 parameters, can be obtained by both distribution functions. It then may be expected that the $\log(N-N_0)$-normal distribution and the 3-parameter Weibull distribution can both agree in a similar way with test data. This is explored in the section below.

3. Fitting distribution functions to test data

Results of similar fatigue tests can be plotted in a normal probability plot at an estimated probability of failure equal to:

$$P_i = \frac{i-0.5}{n} \tag{4}$$

with n as the number of test results and the rank number $i=1$ to n for the fatigue life data in an increasing order of magnitude. Other approximation equations for P_i are

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