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Modified crack-analogy methodology: the equivalent theory

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Abstract

The crack initiation process of fretting fatigue is a complex phenomenon, and various approaches have been instigated to investigate fretting fatigue damage tolerance and crack initiation. The Crack-Analogy Methodology (CAM) is one approach, which is introduced by Giannakopoulos et al. It utilizes the similarity between contact mechanics and fracture mechanics to investigate fretting fatigue life. The present study is motivated by the crack-analogy methodology where a Modified Crack-Analogy Methodology (MCAM) is developed to extend CAM capabilities and to improve its prediction of crack initiation. The modified form extends CAM to include various indenter–substrate geometries as well as modifying its crack initiation parameter to include the effect of the bulk stress in the substrate. MCAM uses the change of the stress intensity factor (ΔK-parameter) as a fretting fatigue crack initiation parameter, since the change is consistent with the cyclic mechanism of fretting fatigue. It uses experimental data to establish ΔK-parameter–life curves similar to the stress–life *S–N* curve prototype in fatigue and to validate crack initiation for various geometric configurations under various load conditions. The results show similar trends to plain fatigue with lower damage tolerance as expected. By including the bulk stress, the crack initiation predictions of fretting fatigue show better consistency with the experimental data than without it where it exhibits fatigue characteristics consistent with *S–N* curves. In general, the ΔK-parameter–life trends show dependency on applied load and pad geometry. Reduced scatter is observed when bulk stress is included in the analyses as compared to that without it. It further demonstrates that the presence of bulk stress in general shifts fretting fatigue characteristics toward low cycle fatigue. The MCAM also predicts the limit of high cycle fatigue. The MCAM formulation shows potentials in life prediction such that it can be used as a tool in the design of components under fretting fatigue.

Keywords: Fretting fatigue; Crack initiation parameter; Fracture mechanics; Contact mechanics

1. Introduction

As the case for fatigue, fretting fatigue in general is a complex subject, where some of the problem modeling fretting fatigue may be characterized as follow [1]:

- Calculations of life are generally less accurate and less dependable than strength calculations. Order of magnitude errors in life estimate are not unusual.
- Fatigue characteristics of a material cannot be deduced from other mechanical properties. They must be measured directly using experiments.
- Full-scale prototype testing is usually necessary to assure an acceptable model predictions.
- Results of different but identical tests may differ widely, requiring, therefore, a statistical interpretation by designer.

Furthermore, damage tolerance of pre-crack initiation in fretting fatigue is of complex nature, which is attributed to its dependency on pad geometries, surface properties, material properties, and mechanical loading conditions [2]. For example, smaller radius cylindrical pads under the same fretting load conditions cause lower damage tolerance (i.e. higher damage) where crack initiates faster than the one with larger radius, which exhibits higher damage tolerance (i.e. lower damage). In the present study, the term crack initiation is used to indicate the number of cycles of pre-crack initiation (i.e. number of cycles leading to the development of small cracks without including the number of cycles of post-crack growth or crack propagation of

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failure). Also, the term damage tolerance is used to quantify the resistance of the material to fretting fatigue damage due to either loading conditions or pad geometry (i.e. higher damage tolerance means more pre-crack initiation compared to lower damage tolerance). Further, the term crack initiation parameter is a norm, where its value is equivalent to an estimated damage tolerance and quantitatively predicts the number of cycles to crack initiation. The parameter is a design tool, which provides a quantitative mean of comparison of material/structure damage under various fretting fatigue scenarios. In practical applications, one needs fretting fatigue crack initiation parameter that reflects its complex process.

Traditionally, the investigation of fretting fatigue crack initiation has been performed using contact mechanics [3–8]. However, the similarity of the stress state near the contact surface using Hertzian contact and asymptotic solution of fracture mechanic has been employed to interpret experimental observations of fretting fatigue crack initiation. For example, the studies [9,10] suggested the similarities between the singular fields for sharp-edged contacts [4–6] and for cracked bodies [11,12]. Nadai [13] analyzed the asymptotic stress solution within an elastic substrate in the vicinity of the corners of the punch for both normal and tangential loading. His result confirms the similarity between the contact surface stress singularity predicted by Sadowski [14], where the elastic singular fields at the sharp edges of two-dimensional contact between a rigid flat punch and planar surface were derived, and the contact surface stresses derived earlier by Hertz [15] for normal loading of a cylindrical punch. Recently, Giannakopoulos et al. [16,17] used the Crack-Analogy Methodology (CAM) to validate and interpret their experimental data. In their studies, a quantitative equivalence has been explored in detail between contact mechanics and fracture mechanics using the vast literature of both fields. This equivalence is established by identifying geometries of cracked bodies, which facilitate an analogy with an axisymmetric sharp-edge pad and rounded pad in contact with flat substrate. Their experimental data are obtained for a spherical pad in contact with substrate. They used a stress intensity factor as crack initiation prediction parameter. They assumed that fatigue crack initiates when the stress intensity factor is greater than the stress intensity threshold. Further, the crack-analogy's results [17] for the weak adhesion case showed that the damage tolerance is insensitive to changes in the tangential load, such that prediction of fretting fatigue crack initiation using mode II stress intensity factor parameter is independent of load, which is contrary to experimental observation. For example, experimental data of cylindrical pads show dependency on radius size and tangential load [18].

One of the fundamental bases of the Modified Crack-Analogy Methodology (MCAM), which is motivated by earlier studies of CAM [16,17], is its utilization of the similarity between contact mechanics and fracture mechanics. This allows the use of fracture mechanic metrics to characterize the contact based fretting fatigue phenomena such as stress intensity factor. MCAM includes the effect of various pad geometry and bulk stress on crack initiation, which is different from CAM. Also, contrary to the stress intensity factor used in CAM, MCAM uses the change of stress intensity factor as fretting fatigue parameter, since it is consistent with the cyclic mechanism of fatigue. It will be referred to as ΔK-parameter for the remaining of the paper. MCAM also establishes ΔK-parameter–life curve similar to S–N curve, which constitutes design information of fundamental importance to specimens subjected to repeated fatigue load.

2. Crack-analogy's methodology

The concept behind the development of MCAM and CAM [16,17] is the equivalence between the asymptotic field of two bodies in contact derived from classical contact mechanics theory and the asymptotic field at a stationary crack determined from fracture mechanics. This allows the use of fracture mechanic metrics to characterize the contact based fretting fatigue phenomena, and it involves the following. First, the identification of the cracked specimen configuration, which provides a geometric equivalence for the contact region of a pad normally pressing on planar substrate, is made, see Fig. 1. Second, its asymptotic solution to the stress and strain fields at the edge of contacts from classical contact is determined. Third, the corresponding stress and strain field solutions for the analogous cracked body from linear elastic fracture mechanics are also determined. In other words, the assumption that the cracked body is subjected to a normal compressive load, whose magnitude is P, and a tangential load, whose magnitude is Q, is the same as that of the load pressing and sliding the pad against the substrate (see Fig. 2). The scalar amplitude of the singular fields at the crack tip is the stress intensity factor. Fourth, equating the different components of the stress fields, one can solve for the stress intensity factor as a function of the load (i.e. an equivalent mode I stress intensity factor for separating or opening is used to characterize the effect of the normal load P, and an equivalent mode II stress intensity factor for slip or shear is used to characterize the effect of the tangential load, Q). Thus, the analogy between the contact mechanics and the fracture mechanics is established, the equivalency of the stress fields can be determined, and MCAM, for example, uses ΔK-parameter as crack initiation parameter of fretting fatigue.

One of the bases of crack-analogy methodology is the singular field that exists in both the contact mechanics and fracture mechanics solutions. Theoretically, it has been demonstrated that under monotonic compressive loading of contact interface, adhesion induces a tensile square root singular stress fields at the edge of contact. Adhesion is used