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Review on critical impact velocities in tension and shear

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Abstract

Adiabatic heating, due to conversion of plastic work into thermal energy, substantially changes the boundary value problems in the theory of plastic wave propagation. Besides a systematic review of the subject, the thermal coupling during plastic wave propagation leading to adiabatic wave trapping is the main subject of this study. Two cases are analyzed, the adiabatic wave trapping in tension and also in shear. The case of shear is relatively new. The wave trapping by adiabatic deformation via thermal softening leads to the so called critical impact velocity (CIV). Theory, experiments and numerical analyses of the CIV in tension and shear is the main part of this paper.

Keywords: Critical impact velocity; Dynamic plasticity; Thermal coupling; Impact shearing

1. Introduction

In the late 1940s of the last century von Kàrman [1–4] and others developed a theory for propagation of one-dimensional plastic waves in a long bar. It was then demonstrated that if an infinite bar is loaded in tension by a sufficiently high impact velocity, plastic deformation is concentrated near the impact end of the bar. The theory was limited to rate independent and isothermal case. However, plastic deformation of materials is rate and temperature dependent. In this paper a more detailed discussion is offered on theory of plastic wave propagation with thermal coupling. The adiabatic heating causes usually a material softening leading to adiabatic wave trapping. Localization of plastic deformation in adiabatic conditions superimposed on inertia effects (waves) causes that the plastic wave speed reaches zero and the critical impact velocity (CIV) occurs. It is shown that the CIV can be observed in both tension and shear. The case of shear has been found and analyzed more recently [5–10].

2. Isothermal propagation of plastic waves (revisited)

Although elastic waves were studied since the beginning of Nineteen Century, for example T.Young in 1807 studied the propagation of elastic strains in a cylindrical bar subjected to tension impact. The theory of plastic waves was formulated in the mid of Twentieth Century by von Kàrman and Taylor, [1,3,4]. The unloading

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elastic waves were introduced by Rakhmatulin [11]. The stress waves caused by an impact at the end of a semi-infinite bar has been analyzed in the case where the impact velocity is high enough to produce plastic deformation.

Consider a bar extending from $x_1 = -\infty$ to $x = 0$ and assume that the end at $x = 0$ is loaded by impact with a constant velocity V_0. The stress–strain relation for the material is given in the form $\sigma(\varepsilon)$, where σ and ε are, respectively, the stress and strain. The $\sigma(\varepsilon)$ relation is unique and has a smooth first derivative $\mathrm{d}\sigma/\mathrm{d}\varepsilon$ in the form of decreasing function of strain. Such assumption leads to the so-called rate-independent theory of plastic wave propagation. It means that initially the rate effects are neglected. However, in the up-to-date approach the existence is accepted of *one and only one* stress–strain relation which may be also a high-strain rate relation at strain rate say $10^3\,\mathrm{s}^{-1}$. In addition, the radial contraction of the material, that is contribution of the radial velocity to the inertia effects is neglected. Under these assumptions, the equation of motion for an element of a slender bar can be written in the form

$$\frac{\partial^2 U_1}{\partial t^2} = C^2(\varepsilon)\frac{\partial^2 U_1}{\partial x_1^2} \quad \text{and} \quad \varepsilon = \varepsilon_\mathrm{e} + \varepsilon_\mathrm{p} \tag{1}$$

in the elastic range $C_0 = (E/\rho)^{1/2}$ and in the plastic range $C_\mathrm{p}(\varepsilon_\mathrm{p}) = (1\,\mathrm{d}\sigma/\rho\,\mathrm{d}\varepsilon_\mathrm{p})^{1/2}$. The waves propagate in the x_1 direction, U_1 is the displacement in that direction, t is time, C_0 is the longitudinal elastic wave speed in slender rods, and $C_\mathrm{p}(\varepsilon_\mathrm{p})$ is the plastic wave speed as a decreasing function of plastic strain ε_p. In the elastic range the wave speed is constant, it depends only on the density ρ and Young's modulus E. Since the boundary conditions are $U_1 = V_0\,t$ for $x_1 = 0$ and $U_1 = 0$ for $x_1 = -\infty$ the solution of Eq. (1) is in the form

$$U_1(x_1, t) = V_0\left[t + \left(\frac{x_1}{C_\mathrm{p}(\varepsilon_\mathrm{p})}\right)\right]. \tag{2}$$

The plastic wave speed has an arbitrary value. The second solution is obtained by putting

$$\left(\frac{\mathrm{d}\sigma}{\rho\,\mathrm{d}\varepsilon_\mathrm{p}}\right)^{1/2} = \pm\frac{x_1}{t} \quad \text{or} \quad C_\mathrm{p}(\varepsilon_\mathrm{p}) = \pm\frac{x_1}{t}. \tag{3}$$

In the above analysis presented previously in [1,4] a special case is considered in which the plastic strain is a function of x_1/t but not of x_1 and t independently.

Since Eq. (1) is a quasi-linear differential equation of the second order, and of the type of the wave equation, it can be also solved by the method of characteristics [12]. The definition of the characteristic line is (x_1 is replaced by x)

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \pm C(\varepsilon) \tag{4}$$

in the elastic range $\mathrm{d}x/\mathrm{d}t = \pm C_0$ and in the plastic range $\mathrm{d}x/\mathrm{d}t = \pm C_\mathrm{p}(\varepsilon_\mathrm{p})$. The partial differential equation (1) satisfied along the characteristic lines due to consistency conditions is

$$\mathrm{d}v = \pm C(\varepsilon)\,\mathrm{d}\varepsilon. \tag{5}$$

Along two sets of characteristics the mass velocities in both elastic and plastic ranges are given by

$$v = \pm C_0\varepsilon \quad \text{and} \quad v(\varepsilon_\mathrm{p}) = \pm\int_0^{\varepsilon_\mathrm{p}} C_\mathrm{p}(\xi)\,\mathrm{d}\xi. \tag{6}$$

The first relation in (6) occurs along the linear characteristics $\mathrm{d}x/\mathrm{d}t = \pm C_0$. The mass velocity $v(\varepsilon_\mathrm{p})$ defined by the second relation in (6), that is in the plastic range, occurs along non-linear characteristics $\mathrm{d}x/\mathrm{d}t = \pm C_\mathrm{p}(\varepsilon_\mathrm{p})$. In a more general approach, applied nowadays, the stress–strain relation is assumed at constant strain rate, typical value $\sim 10^3\,\mathrm{s}^{-1}$, and the temperature is assumed as the initial temperature. The generalized wave speed is given by

$$C_\mathrm{p}(\varepsilon_\mathrm{p}) = \pm\left(\frac{1}{\rho}\frac{\mathrm{d}\sigma}{\mathrm{d}\varepsilon_\mathrm{p}}\right)^{1/2}_{\dot\varepsilon, T}. \tag{7}$$