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On stability prediction for milling

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Abstract

Stability of 2-dof milling is investigated. Stability boundaries are predicted by the zeroth order approximation (ZOA) and the semi-discretization (SD) methods. While similar for high radial immersions, predictions of the two methods grow considerably different as radial immersion is decreased. The most prominent difference is an additional type of instability causing periodic chatter which is predicted only by the SD method. Experiments confirm predictions of the SD method, revealing three principal types of tool motion: periodic chatter-free, quasi-periodic chatter and periodic chatter, as well as some special chatter cases. Tool deflections recorded during each of these motion types are studied in detail.

Keywords: End milling; Stability prediction

1. Introduction

High material removal rates, provided in theory by the modern machining centers, often cannot be achieved in practice due to the inherent instability of a cutting process. In cutting processes which involve rotation of the tool or workpiece, the instability is caused by the so called regeneration of surface waviness during successive cuts: wavy surface left behind by the previous cut influences chip thickness during the current cut, thereby contributing to the wavy surface, which in turn influences chip thickness in the successive cut, etc. The resulting instability is called regenerative chatter.

Dynamics of regenerative cutting processes can be described by models in the form of linear delay-differential equations (DDEs) [1,2]. Chatter-free cutting and chatter correspond respectively to the linearly stable and unstable solutions of the model equations. The cutting parameters that assure stable, chatter-free machining can therefore be predicted by the linear stability analysis of the equations. The stability boundary is usually presented in a graph of the maximal chatter-free depth of cut vs. spindle speed. The graph is called a stability chart.

For continuous cutting, such as uninterrupted turning, the model equations are autonomous and the stability boundary can be given in closed form. The stability boundary has a typical 'lobed' structure, with stability maxima located at spindle speeds corresponding to the integer fractions of the eigen-frequencies of the most flexible modes of the machine–tool–workpiece system. The instability, i.e. the transition from chatter-free cutting to chatter, corresponds to the sub-critical Hopf bifurcation [3,4].

For interrupted machining, such as milling and interrupted turning, the cutting force variation is time-periodic. The resulting model equations are non-autonomous DDEs for which the linear stability condition cannot be given in closed form. The stability boundary can be determined numerically, by time domain simulations [5–7]. While indispensable for the study of complicated cutting process models which involve trochoidal tool path, multiple regeneration effects, non-linear force dependencies, etc., the time domain simulations are an inefficient way of

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exploring the parameter space of linear DDEs. The first analytical attempts at stability prediction of milling were based on Fourier expansion of the time-periodic force coefficient [8,9]. The accuracy of the obtained stability boundary depends on the shape of the cutting force variation and the number of Fourier terms used to approximate it. For cutters with a large number of teeth and for substantial radial immersions, the cutting force varies relatively little so that reasonably accurate stability predictions can be achieved by using only the zeroth order Fourier term [10,11]. However, for cutters with few teeth and for low radial immersions, prohibitively many Fourier terms may be needed to capture the cutting force variation. In such cases, which are quite common in high-speed milling, the exact stability boundary may differ significantly from the one predicted by the zeroth order approximation (ZOA).

This discrepancy was first shown for the case of very low immersion milling, where the time interval of tool–workpiece contact was a small fraction of the spindle period [12,13]. Based on a discrete representation of the impact-like cutting process combined with the exact analytical solution of the free tool vibration, a time-domain analytical method for stability prediction was developed which revealed that there exist two types of instability in milling: the Hopf bifurcation, which causes the quasi-periodic chatter, and the period doubling or flip bifurcation, which causes the periodic chatter. The stability boundary in milling therefore consists of two sets of lobes corresponding to the two instability types. In contrast, the ZOA method predicts only one type of instability, the Hopf bifurcation.

The stability predictions from Refs. [12,13] lose accuracy as the time of the tool–workpiece contact increases. Two alternative methods have since been proposed that can predict stability boundary for an arbitrary time in the cut. The first method combines the exact analytical solution of the free tool vibration with the approximate solution for the tool vibration during cutting calculated using temporal finite element analysis (TFEA) [14]. The second method employs the semi-discretization (SD) scheme to transform the DDE into a series of autonomous ordinary differential equations (ODEs) for which the solutions are known [15]. In both methods, stability is determined by the eigen-values of the transition matrix which connects the solutions between successive cuts. The methods have been derived and verified experimentally using a 1-dof milling system [16,17]. Recently, both methods have also been extended to 2-dof milling systems [18–20].

In this paper, stability of a 2-dof milling system is investigated. Stability boundaries are predicted using the ZOA and SD methods. The ZOA method is briefly reviewed and the SD method for 2-dof systems is presented. Stability predictions of the two methods are compared for a series of radial immersions and verified experimentally on a high-speed milling center using a long and slender tool. The recorded tool deflections in the X–Y plane are analyzed in detail and six different types of tool motion are distinguished: three principal types predicted by the SD method (periodic chatter-free, quasi-periodic and periodic chatter) as well as three special chatter cases.

2. Stability prediction for end milling

Consider a 2-dof end milling operation shown schematically in Fig. 1. A cutter with a diameter D and N equally spaced teeth rotates at a constant angular velocity Ω. The radial immersion angle of the j-th tooth varies with time as: $\phi_j(t) = \Omega t + 2\pi(j-1)/N$. A compliant machine–tool structure is excited by the tangential (F_t) and radial (F_r) components of the milling force at the tool tip causing dynamic response of the structure governed by the following equation:

$$\boldsymbol{M}\ddot{\boldsymbol{X}}(t) + \boldsymbol{C}\dot{\boldsymbol{X}}(t) + \boldsymbol{K}\boldsymbol{X}(t) = \boldsymbol{F}(t). \tag{1}$$

Here, $\boldsymbol{X}(t)$ and $\boldsymbol{F}(t)$ denote the displacement and cutting force vectors, while $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ denote the system mass, damping and stiffness matrices. For a system with m vibration modes in X and Y directions, the vectors are $2m\times 1$ and the matrices $2m\times 2m$ dimensional. The matrices are diagonal if the modes in X and Y directions are uncoupled.

The feed (F_x) and normal (F_y) cutting force components acting on the j-th tooth are given by:

$$\begin{aligned} F_{x,j}(t) &= [-F_{\mathrm{t},j}(t)\cos\phi_j(t) - F_{\mathrm{r},j}(t)\sin\phi_j(t)]g_j(t), \\ F_{y,j}(t) &= [F_{\mathrm{t},j}(t)\sin\phi_j(t) - F_{\mathrm{r},j}(t)\cos\phi_j(t)]g_j(t), \end{aligned} \tag{2}$$

where $g_j(t)$ represents a unit step function determining whether or not the j-th tooth is cutting. The tangential and radial cutting force components are assumed proportional to the chip load defined by the chip thickness $h_j(t)$ and axial

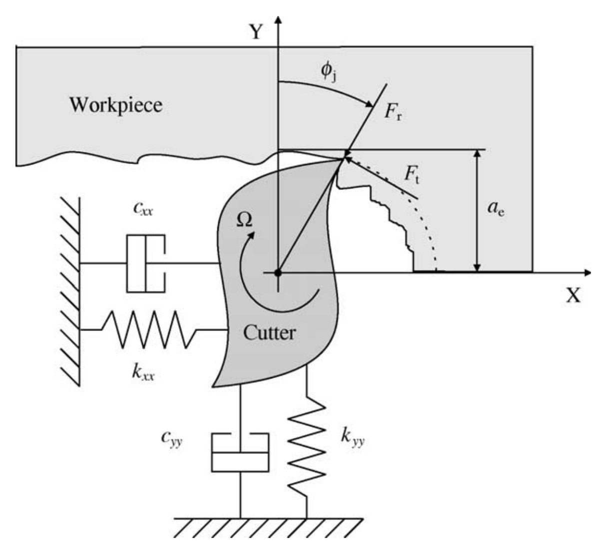

Fig. 1. Schematic diagram of end milling.