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An exact solution for functionally graded piezoelectric laminates in cylindrical bending

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Abstract

In the present paper, the exact solutions of a simply supported functionally graded piezoelectric plate/laminate under cylindrical bending are derived. With similar derivation procedure as that used for Stroh formalism, the eigenrelation and general solutions for the problems can be expressed in very concise forms, which are convenient for further treatments of both analytic and numerical studies. The exact solutions can be served as benchmarks to verify and improve various approximate theories and numerical methods. To show the influence of material gradients, numerical examples based on the exact solutions are given, and some properties of the mechanical and electric responses of the plates under mechanical and electrical forces are discussed.

Keywords: Functionally graded piezoelectric material; Plate; Laminate; Cylindrical bending; Exact solution

1. Introduction

Piezoelectric laminates, such as bimorphs or stacking structures, have been widely used as components of transducers, actuators and sensors due to their inherent electromechanical coupling behaviour [1]. For a piezoelectric laminate with homogeneous material properties in

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layers, high stress concentrations are usually presented at the layer interfaces under mechanical or electric loading, which may lead to lifetime limitations. To reduce the drawbacks, piezoelectric materials and structures with functionally graded material properties (FGM) along layer-thickness direction have been introduced and fabricated [2–5]. For optimal design and fabrication of a functionally graded piezoelectric material (FGPM) with desired properties, predicting and understanding the relationship between its compositional gradient and electromechanical response is of primary importance, which has attracted researchers' increasing attentions.

According to the literature, most of the studies on FGPM laminates were based on various 2-D plate theories, except some studies dealing with crack problems. It is known that most of the 2-D laminate theories belong to equivalent single-layer (ESL) theories, in which the distributions of mechanical and electrical properties through the layer-thickness have been assumed in advance. The ESL plate theories are therefore suitable for predicting resultant electromechanical response of a piezoelectric laminate. For a FGPM laminate with specific interest on the variations of electromechanical response across the thickness, 3-D based theories may be better choices than the ESL plate theories. In particular, some 3-D exact solutions for FGPM structures are essential for the purpose of verifying and improving related approximate theoretical and numerical methods. It seems however that very few exact solutions on FGPM plates and laminates have been reported [6–9].

In this paper, the exact solutions of a simply supported FGPM plate/laminate under cylindrical bending are derived. This investigation has not been addressed although exact analysis of homogeneous piezoelectric plates/laminates under cylindrical bending has been well-studied [10–13]. In the derivation, the treatment used in [13,14] to express the solutions in the form similar to the Stroh formalism is extended. By comparing with the solutions obtained by state space approach [8], the eigenrelations and the general solutions solved by the present method are more concise and elegant. To show the influence of material gradients, numerical examples based on the exact solutions are also given and discussed in the paper.

2. Basic equations

Consider a laminate composed either completely or in part of functionally graded piezoelectric layers, as shown in Fig. 1. The laminate is of length L in the x_2-direction, and is of total thickness h in the x_3-direction. The two edges $x_2 = 0$ L of the laminated plate are simply supported, and the plate is under cylindrical bending.

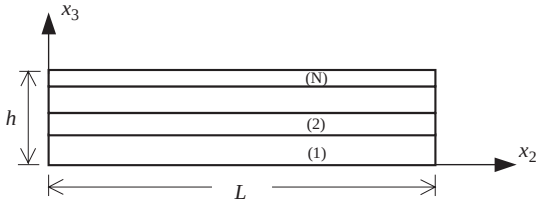

Fig. 1. Geometry of a laminate in cylindrical bending.