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On statistical quasi-linearization

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Abstract

In carrying out the statistical linearization procedure to a non-linear system subjected to an external random excitation, a Gaussian probability distribution is assumed for the system response. If the random excitation is non-Gaussian, however, the procedure may lead to a large error since the response of bother the original non-linear system and the replacement linear system are not Gaussian distributed. It is found that in some cases such a system can be transformed to one under parametric excitations of Gaussian white noises. Then the quasi-linearization procedure, proposed originally for non-linear systems under both external and parametric excitations of Gaussian white noises, can be applied to these cases. In the procedure, exact statistical moments of the replacing quasi-linear system are used to calculate the linearization parameters. Since the assumption of a Gaussian probability distribution is avoided, the accuracy of the approximation method is improved. The approach is applied to non-linear systems under two types of non-Gaussian excitations: randomized sinusoidal process and polynomials of a filtered process. Numerical examples are investigated, and the calculated results show that the proposed method has higher accuracy than the conventional linearization, as compared with the results obtained from Monte Carlo simulations.

Keywords: Random vibration; Quasi-linearization; Non-Gaussian excitations; Statistical moments; Spectral density

1. Introduction

The method of statistical linearization has been widely used in treating non-linear systems under external random excitations [1–6]. In the method, the original non-linear system is replaced by an equivalent linear system with its linearization coefficients determined by a statistical criterion, such as the least mean-square difference. When carrying out the procedure, a Gaussian probability distribution is usually assumed for the system response. The assumption is in fact another approximation besides the replacement of the non-linear system since (i) the response of a non-linear system is generally non-Gaussian even if the excitation process is Gaussian, and (ii) the response of the equivalent linear system is also non-Gaussian if the excitation process is non-Gaussian. To improve the accuracy, non-Gaussian probability distributions have been suggested to perform the statistical linearization for some types of non-linear systems [7–9]. It is found

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that the accuracy is indeed improved if appropriate non-Gaussian distributions are adopted. However, selection of such a non-Gaussian distribution for a specific non-linear system is challenging since it is generally unknown.

It has been found that systems under some types of non-Gaussian random excitations can be transformed to the systems under parametric excitations of Gaussian white noises. As a price for the simplification from non-Gaussian excitations to Gaussian white noises, the dimension of the system is increased. Nevertheless, the advantage of the transformation lies in the applicability of the quasi-linearization approach proposed in [10] to the new transformed systems. The quasi-linearization procedure was originally proposed for non-linear systems subjected to both external and parametric excitations of Gaussian white noises. In the procedure, such a non-linear system is replaced by an equivalent quasi-linear one with the excitation terms kept unchanged. The coefficients in the quasi-linear system are determined by using the statistical moments which can be solved exactly for the replacing quasi-linear system; thus, the assumption of Gaussian distribution for the system response is avoided. In the present paper, the quasi-linearization approach is applied to non-linear systems subjected to two types of non-Gaussian random excitations: randomized sinusoidal process and polynomials of filtered process. The common features of the two cases are (i) the probability distributions of the excitations as well as of the system responses may be highly non-Gaussian; (ii) the excitations can be modeled in terms of Gaussian white noises. Numerical examples are investigated, and results obtained from the proposed quasi-linearization, the conventional linearization, as well as Monte Carlo simulations are compared.

2. Quasi-linearization

To proceed, a brief review of the quasi-linearization is first given below, following [10]. Consider a system governed by the following equations:

$$\ddot{Y}_j+h_j(\boldsymbol{Y},\dot{\boldsymbol{Y}})=\sum_{i=1}^{n}[\alpha_{ji}Y_iW_{1i}(t)+\beta_{ji}\dot{Y}_iW_{2i}(t)]+W_{3j}(t),\quad (j=1,2,\ldots,n), \tag{1}$$

where $h_j(\boldsymbol{Y},\dot{\boldsymbol{Y}})$ is a non-linear function including both damping and stiffness forces, and $W_{1i}(t)$, $W_{2i}(t)$ and $W_{3j}(t)$ are Gaussian white noises. The first step in the quasi-linearization procedure is to replace the non-linear forces in $h_j(\boldsymbol{Y},\dot{\boldsymbol{Y}})$ by linear forces, leading to

$$\begin{aligned}\ddot{Y}_j &+ \sum_{i=1}^{n}(a_{ji}Y_i+b_{ji}\dot{Y}_i)\\ &=\sum_{i=1}^{n}[\alpha_{ji}Y_iW_{1i}(t)+\beta_{ji}\dot{Y}_iW_{2i}(t)]+W_{3j}(t),\\ &(j=1,2,\ldots,n).\end{aligned} \tag{2}$$

The system described by (2) is said to be quasi-linear since the principle of superposition is not applicable due to the presence of the multiplicative excitations. Letting $Y_j=X_j$ and $\dot{Y}_j=X_{j+n}$, a set of Ito stochastic differential equations [11] is derived from (2) as follows:

$$\begin{aligned}\mathrm{d}X_j &= X_{j+n}\,\mathrm{d}t,\\ \mathrm{d}X_{j+n} &= \sum_{i=1}^{2n}C_{ji}X_i\,\mathrm{d}t+\sum_{i=1}^{n}\sigma_{ji}(\boldsymbol{X})\,\mathrm{d}B_i(t),\\ &(j=1,2,\ldots,n),\end{aligned} \tag{3}$$

where $B_i(t)$ are unit Wiener processes, and C_{ji} and $\sigma_{ji}(\boldsymbol{X})$ are derived from (2) by incorporating the Wong-Zakai correction terms, (see e.g. [12]). It is known from (2) that C_{ji} are constants in the present case.

To find the equivalent linear coefficients a_{ji} and b_{ji} in (2), the following mean-square differences are minimized

$$E\left\{\left[h_j(\boldsymbol{Y},\dot{\boldsymbol{Y}})-\sum_{i=1}^{n}(a_{ji}Y_i+b_{ji}\dot{Y}_i)\right]^2\right\},\quad (j=1,2,\ldots,n) \tag{4}$$

which leads to

$$E[\boldsymbol{X}\boldsymbol{X}']\begin{bmatrix}\boldsymbol{a}'\\ \boldsymbol{b}'\end{bmatrix}=E[\boldsymbol{X}\boldsymbol{h}'], \tag{5}$$

where a prime denotes a matrix transposition, $\boldsymbol{X}=\{X_1\ X_2\ \ldots\ X_{2n}\}'$, $\boldsymbol{a}=[a_{ij}]$, $\boldsymbol{b}=[b_{ij}]$, and $\boldsymbol{h}=\{h_1\ h_2\ \ldots\ h_n\}'$. The terms in $E[\boldsymbol{X}\boldsymbol{X}']$ on the left-hand side of (5) are the second-order moments of the state variables. Assuming that functions $h_j(\boldsymbol{Y},\dot{\boldsymbol{Y}})$ are polynomials of Y_j and $\dot{Y}_j$, which is true for many