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International Journal of Non-Linear Mechanics 40 (2005) 465–474

INTERNATIONAL JOURNAL OF NON-LINEAR MECHANICS

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Flow and heat transfer of an electrically conducting fluid of second grade in a porous medium over a stretching sheet subject to a transverse magnetic field

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Received 6 January 2004; received in revised form 28 May 2004

Abstract

An analysis is performed for flow and heat transfer of a steady laminar boundary layer flow of an electrically conducting fluid of second grade in a porous medium subject to a transverse uniform magnetic field past a semi-infinite stretching sheet with power-law surface temperature or power-law surface heat flux. The effects of viscous dissipation, internal heat generation of absorption and work done due to deformation are considered in the energy equation. The variations of surface temperature gradient for the prescribed surface temperature case (PST) and surface temperature for the prescribed heat flux case (PHF) with various parameters are tabulated. The asymptotic expansions of the solutions for large Prandtl number are also given for the two heating conditions. It is shown that, when the Eckert number is large enough, the heat flow may transfer from the fluid to the wall rather than from the wall to the fluid when Eckert number is small. A physical explanation is given for this phenomenon.

Keywords: Second grade fluid; Stretching sheet; Porous medium; Magnetic field

1. Introduction

The studies of laminar boundary layer flows of non-Newtonian fluids have received much attention because the power needed in stretching a sheet and the heat transfer rate in a non-Newtonian fluid are quite different from those of a Newtonian fluid. The boundary layer behavior on a moving continuous solid surface is an important type of flow arising in many engineering processes. For example, materials manufactured by extrusion process and heat-treated materials traveling between a feed roll and a wind-up roll or on conveyor belts possess the features of a moving continuous surface. Thus the study of boundary layer flow of a viscoelastic fluid has been the main subject of a large number of researches in the past [1–5].

Recently the applications of the viscoelastic boundary layer flow extrude to the area with additional

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doi:10.1016/j.ijnonlinmec.2004.07.008

effects, such as the heat transfer in a porous medium and the effects of magnetic field, variable viscosity, and diffusion of chemical reactive species, etc. [6–12].

A viscoelastic fluid is termed as second grade if the Cauchy stress tensor $\mathbf{T}$ is given by

$$\mathbf{T} = -p\mathbf{I} + \mu\mathbf{A_1} + \alpha_1\mathbf{A_2} + \alpha_2\mathbf{A_1^2}, \tag{1}$$

where p is the indeterminate pressure constrained by the incompressibility, μ is the viscosity, α_1 and α_2 are the moduli of the second grade fluid, and $\mathbf{A_1}$ and $\mathbf{A_2}$ are the first two Rivlin-Ericksen tensors [13] defined by

$$\begin{aligned} &\mathbf{A_1} = \mathbf{L} + \mathbf{L^T}, \\ &\mathbf{A_2} = \frac{\mathrm{d}\mathbf{A_1}}{\mathrm{d}t} + \mathbf{A_1 L} + \mathbf{L^T A_1}, \\ &\mathbf{L} = \nabla\mathbf{V}, \end{aligned} \tag{2}$$

where $\mathrm{d}/\mathrm{d}t$ is the material derivative. If the fluid of second grade is to satisfy the Clausius-Dehum inequality for all motions and the assumption that the specific Helmholtz free energy of the fluid is a minimum when it is locally at rest, then the requirements for the moduli of the second grade fluid are

$$\mu \geqslant 0, \quad \alpha_1 > 0, \quad \text{and} \quad \alpha_1 + \alpha_2 = 0.$$

Though the sign of α_1 has been a subject of much controversy, we do not intend to discuss it since a critical review of Dunn and Rajagopal [14] has already given a concise discussion about this issue.

The flow problem of non-Newtonian fluids, characterized by Bingham plastic and the power law models, in a magnetic field has been investigated by Sarpkaya [15]. Sarpkaya [15] also pointed out that some non-Newtonian fluids such as nuclear fuel slurries, liquid metals, mercury amalgams, biological fluids, plastic extrusions, paper coating, lubrication oils and greases, have applications in many areas in the absence as well as in the presence of a magnetic field. Char [16] studied the heat and mass transfer in a hydromagnetic flow of a viscoelastic fluid, the Walters' B liquid, over a stretching sheet, however, only the thermal diffusion is considered in the energy equation.

If an incompressible second grade fluid in a porous medium is electrically conducting, the Lorentz force must be taken into account in the momentum equations when a transverse uniform magnetic field is applied to the fluid layer. The governing equations are the continuity equation and momentum equation with negligible gravitational body force and they are

$$\nabla \cdot \mathbf{V} = 0, \tag{3}$$

$$\rho\frac{\mathrm{d}\mathbf{V}}{\mathrm{d}t} = \nabla\mathbf{T} + \sigma(\mathbf{V}\times\mathbf{B})\times\mathbf{B} - \frac{\mu}{k_1}\mathbf{V}, \tag{4}$$

where $\mathbf{V}$ is the velocity vector, σ is the electrical conductivity of the fluid, k_1 is the permeability, $\mathbf{B} = (0, B_0, 0)$ is the applied uniform magnetic field. Here the induced magnetic field is neglected if the magnetic Reynolds number is very small [17]. Besides, the electric field is assumed to be zero. It should be noted that the viscosity in the Darcy's term of (4) may be not equal to the effective viscosity in the Cauchy stress tensor, however, we assume that they are the same for simplicity [18].

The inclusion of the effect of magnetic field on an electrically conducting second grade fluid in a porous medium shows that it will change the distribution of velocity, however, the solution of the velocity retains its form as obtained by Troy et al. [19] for the problem of a viscoelastic fluid. We give the closed form solutions of this boundary layer problem of second grade fluid saturated in a porous medium with $\alpha_1 > 0$ for power law surface temperature and surface heat flux conditions when the effects of viscous dissipation, work done by deformation and internal heat generation or absorption are considered in the energy equation. It is shown that the heat transfer may be in the direction from the fluid to the wall when Eckert number is large enough rather than from the wall to the fluid when Eckert number is small. A physical explanation is presented for this phenomenon.

2. Flow analysis

Consider an incompressible, electrically conducting fluid of second grade, obeying Eqs. (1) and (2), through the porous medium subject to a transverse magnetic field over a stretching sheet with the plane $y = 0$, then the fluid is occupied above the sheet $y > 0$. Two equal and opposite forces are introduced along the x-axis so that the sheet is stretched keeping the origin fixed. Substituting (1) and (2) into (3) and (4) and using the usual boundary layer assumption,