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Lower bound shakedown analysis by the symmetric Galerkin boundary element method

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Abstract

In this paper, the static shakedown theorem is reformulated making use of the symmetric Galerkin boundary element method (SGBEM) rather than of finite element method. Based on the classical Melan's theorem, a numerical solution procedure is presented for shakedown analysis of structures made of elastic-perfectly plastic material. The self-equilibrium stress field is constructed by linear combination of several basis self-equilibrium stress fields with parameters to be determined. These basis self-equilibrium stress fields are expressed as elastic responses of the body to imposed permanent strains obtained through elastic–plastic incremental analysis. The lower bound of shakedown load is obtained via a non-linear mathematical programming problem solved by the Complex method. Numerical examples show that it is feasible and efficient to solve the problems of shakedown analysis by using the SGBEM.

Keywords: C. SGBEM; B. Lower bound; B. Shakedown analysis; C. Non-linear programming; C. Complex method

1. Introduction

The design of engineering structures subjected to variable external loads demands a realistic evaluation of the safety bound with respect to failure. But traditional

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linear elastic analysis always makes conservative results of engineering problems so that the load-carrying capacities of the structures cannot be brought into play effectively. For this reason, the concept of safety based on failure load has increasingly gained importance in the last decades. In fact, in many engineering situations in the presence of variable repeated actions, economy of design requires to admit plastic deformations, or these turn out to be inevitable anyway. Using elastic–plastic analysis for structural design and safety evaluation can make the computational results more economical. So, elastic–plastic analysis method is more and more widely applied to engineering problems.

The elastic–plastic response to variable repeated actions may imply structural failure either by low-cycle fatigue or by incremental collapse. The former phenomenon consists of alternating plastic strains which eventually cause local material failure; the latter is of rather global nature, consists of a gradual divergence of the deformed configuration and eventually results in local failure or in excessive displacements causing inserviceability or loss of stability (König and Maier, 1981).

As a simplified method, shakedown analysis has higher computational efficiency and is more practical than incremental analysis (Pham, 2001; Zhang and Raad, 2002). Theoretically this method can avoid the elastic–plastic incremental computation which is usually time-consuming, but on the other hand, it faces great difficulty in numerical computation. With solution procedures, it is mostly centered on mathematical programming (Cohn et al., 1979; Maier and Munro, 1982; Liu et al., 1995; Corradi and Vena, 2003). Because this mathematical programming has excessive independent variables and constraint conditions, and in general is a nonlinear programming, so the scale of solving is quite large. At present, many scholars are making great efforts to develop efficient computational methods of shakedown analysis (Pham, 2003; Bousshine et al., 2003); most of these works aim at overcoming this difficulty.

Most works on numerical shakedown analysis are based on the finite element method (FEM). As an important alternative to the finite element method, the boundary element method (BEM) has rapidly developed and found many applications in engineering (Liu et al., 1997; Maier et al., 1993). Shakedown analysis was first addressed by the boundary element method (Maier and Polizzotto, 1984), where the direct method was employed for the formulation of the problem for computing the shakedown load either through the static approach or through the kinematic approach. It was shown that the problem is so reduced to one of mathematical programming as in the case of the finite element method. Teixeira de Freitas (1991) proposed a kinematic model for plastic limit analysis of solids by the boundary integral method. Shakedown analysis was also addressed by making use of the Galerkin BEM (Panzeca, 1992; Panzeca et al., 2000). It was shown that the Galerkin BEM enables one to obtain mathematical programming problems for the shakedown load computations that possess symmetry and sign definiteness properties as occurs within the FEM context. However, up to now, to the authors' knowledge, no computational results have been given in these papers.

In this study, our attention is focused on the symmetric Galerkin BEM solution procedure for lower bound shakedown analysis. The considered solid is made up of