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Modelling of elliptical cracks in an infinite body and in a pressurized cylinder by a hybrid weight function approach

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Abstract

In this work, a hybridization technique is proposed. It consists of using two weight functions to model elliptical cracks for computation of the stress intensity factor 'SIF' in mode I. The idea of hybridization consists of dividing the ellipse into two zones, then to use each weight function in the area where it is more efficient. The proportion between the two zones is determined by optimization of the ellipse axis ratio. A computer code is developed for the computation of SIF. The treatment of the numerical procedures including singularities are presented in detail. The approach is tested on several applications (elliptical crack in infinite body, semi-elliptical cracks in thin and thick cylinders), to demonstrate its accuracy by minimization of the error of SIF and its correlation with respect to other researchers.

Keywords: Pressurized cylinder; Stress intensity factor; Elliptical crack; Hybridization; Weight function

1. Introduction

The development of weight functions in fracture mechanics started with the work of Bueckner [1], based on the formulation by the Green's function, for a semi-infinite crack, in an infinite medium.

The investigation of weight functions and the evaluation of the energy balance formula of Rice [2], allowed the extension of the use of weight functions by several authors such as Oore and Burns [3] and Bortmann and Banks-Sills [4]. Gao and Rice [5] introduced a study of a semi-infinite crack front during its coplanar propagation, from which results the values of stress intensity factor (SIF) along the crack front. Recently, Sun and Wang [6] gave in-depth interpretations of the energy release rate of the crack front. Other investigations have related to crack shape (ellipse, half of ellipse, quarter of ellipse, rectangle…), to the mode of rupture (mode I, II, III or mixed), and to the large domain of application (elastoplastic, elastodynamic, thermoelastic…). Among those works, one can chronologically mention, Fett et al. [7], Vainshtok et al. [8], Dominguez et al. [9], Rooke et al. [10], Orynyak et al. [11], Zheng et al. [12], Kiciak et al. [13], Pommier et al. [14], Krasowsky et al. [15], Hachi et al. [16], and Christopher et al. [17]. The principle of the weight function technique consists in employing one or more known solutions (known as reference solutions) of a particular case in order to find the general solution. The reference solution generally comes from analytical results (exact). However, in some cases, the absence of such results obliges authors, such as Orynyak et al. [11], Zheng et al. [12], Kiciak et al. [13], Pommier et al. [14], and Christopher et al. [17], to use approximate solutions which could be existing weight functions. The solution of the SIF in mode I using the weight function technique is given by the general form [11]

$$K_{IQ'} = \int_{(S)} W_{QQ'} q(Q)\, dS \qquad (1)$$

where $K_{IQ'}$ is the stress intensity factor in mode I at the (Q') point on the crack front. $W_{QQ'}$ is the weight function related to the problem, and defined as $K_{IQ'}$ which is generated by a unit concentrated and symmetrical force applied to

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the arbitrary Q point of the crack, and $q(Q)$ is the applied load at Q.

2. Presentation of the hybridization technique

Our study is based on the hybridization of two weight functions. The first one was developed by Oore and Burns [3] to model any closed shape of crack in an infinite body, including elliptical cracks. Its expression is as follows

$$W_{QQ'} = \frac{\sqrt{2}}{\pi l_{QQ'}^2 \sqrt{\int_\Gamma \frac{d\Gamma}{(\rho_Q)^2}}} \tag{2}$$

with r and ϕ the polar coordinates of an arbitrary point Q. $l_{QQ'}$ is the distance between the (Q') point and the arbitrary Q point. (Γ) is the curve of the ellipse (the crack front), and ρ_Q is the distance between the Q point and the elementary segment $d\Gamma$.

The second one was developed by Krasowsky et al. [15] to model elliptical cracks in an infinite body. Its expression is as follows

$$W_{QQ'} = \frac{2\Pi^{1/4}(\theta)}{\sqrt{\pi a\left(1-\frac{r^2(\varphi)}{R^2(\varphi)}\right) l_{QQ'}^2 \int_\Gamma \frac{d\Gamma}{(\rho_Q)^2}}} \tag{3}$$

with $\Pi(\theta) = (\sin^2\theta + \alpha^4\cos^2\theta)/(\sin^2\theta + \alpha^2\cos^2\theta)$ and $\alpha = a/b$.

The principle of hybridization is to divide the elliptical crack into two zones, an internal zone I and an external zone II (see Fig. 1), then to use each of the two weight functions in the area where it is more efficient.

The weight function of Eq. (3) is intended exclusively for cracks of elliptical form. Nevertheless, it presents an additional singularity $(1-r/R)^{-1/2}$ compared to Eq. (2). This makes Eq. (3) less efficient in the vicinity of the crack front ($r \to R$). This argument leads us to choose the weight function (3) for zone I, and the weight function (2) for zone II, where the singularity $(1-r/R)^{-1/2}$ is very strong.

The hybridization verifies well the two geometrical cases of an elliptical crack $\alpha \to 0$ and $\alpha \to 1$, as long as the two

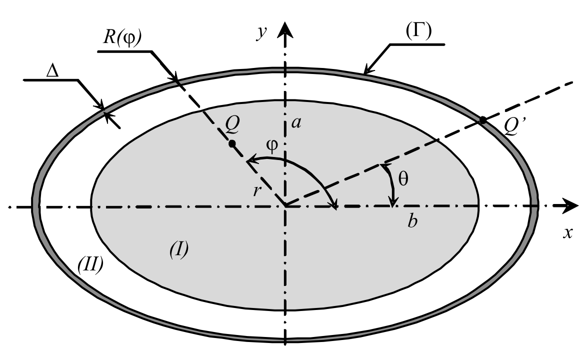

Fig. 1. Subdivision of the elliptical crack in two zones.

functions (2) and (3) satisfy the two following conditions

$$\underset{\alpha \to 0}{W_{QQ'}} \to W_{QQ'}(\text{straight}) = \frac{\sqrt{2d}}{\pi\sqrt{\pi} l_{QQ'}^2} \tag{4}$$

is the weight function of a straight line crack in a semi-infinite body [15]. d is the shortest distance between the crack front and the arbitrary Q point

$$\underset{\alpha \to 1}{W_{QQ'}} \to W_{QQ'}(\text{penny-shaped}) = \frac{\sqrt{R^2-r^2}}{\pi\sqrt{\pi R} l_{QQ'}^2} \tag{5}$$

is the weight function of a penny-shaped crack in an infinite body [15] (see Fig. 2).

In Fig. 2, the parameters f_1 and f_2 are defined as

$$f_1 = \int_S (W_{QQ'})_{\text{eqn}(2)} dS \Big/ \int_S (W_{QQ'})_{\text{eqn}(4)} dS \quad \text{and}$$

$$f_2 = \int_S (W_{QQ'})_{\text{eqn}(3)} dS \Big/ \int_S (W_{QQ'})_{\text{eqn}(4)} dS.$$

The proportion between the two zones is established by optimization for the given aspect ratio α. For that, two remarks are to be pointed out:

- For any value of θ, when $\alpha \to 1$, the weight function (2) tends more quickly to the weight function (4) than does the weight function (3). This has been numerically checked (see Fig. 2 for $\theta = 0°$).
- The weight function (3) is adapted to large changes of the radius of curvature along the crack front This is confirmed by the presence of the radius of curvature, which equals $(a/\alpha)(\Pi(\theta))^{3/2}$.

The above two remarks allow us to conclude the following: the smaller is α, the larger is zone I and vice versa. Thus, we adopted the following convention:

$$\text{For } r/R \le 1-\alpha^2 \quad \text{then} \quad W_{QQ'} = W_{QQ'} \text{ of eqn (3)} \tag{6a}$$

and

$$\text{for } r/R > 1-\alpha^2 \quad \text{then} \quad W_{QQ'} = W_{QQ'} \text{ of eqn (2)} \tag{6b}$$

3. Numerical implementation

According to Eq. (1), the solution includes two integrals to be considered. A surface integral and a curvilinear integral (contour integral) of the crack front (Γ) included in the two functions (2) and (3).

3.1. The surface integral

Treatment of the singularity $1/l_{QQ'}^2$ in Eqs. (2) and (3) is addressed by Krasowsky et al. [15]. This consists of