Available online at www.sciencedirect.com
SCIENCE DIRECT®

International Journal of Pressure Vessels and Piping 82 (2005) 770–776

INTERNATIONAL JOURNAL OF
Pressure Vessels and Piping

www.elsevier.com/locate/ijpvp

Limit analysis based on a modified elastic compensation method for nozzle-to-cylinder junctions

P. Yang[a], Y. Liu[a,*], Y. Ohtake[b], H. Yuan[a], Z. Cen[a]

[a]Department of Engineering Mechanics, Tsinghua University, Beijing 100084, China
[b]Ishikawajima-Harima Heavy Industries Co. Ltd., Yokohama, Japan

Received 14 January 2004; revised 13 June 2005; accepted 13 June 2005

Abstract

To address the computational difficulties of the elastic compensation method (ECM) for complex structures such as nozzle-to-cylinder junctions, this paper develops a modified elastic compensation method (MECM). This method improves the precision of the ECM while preserving the advantages such as simplicity and high efficiency. Limit loads are calculated for three representative examples. The calculated solutions are compared with results from the elastic-plastic analysis method and the twice-elastic slope method. It is found that the MECM can provide a good estimation of plastic limit loads for complex structures.

Keywords: Modified elastic compensation method; Elastic modulus; Limit load; Nozzle-to-cylinder juncture

1. Introduction

Standard evaluation rules for pressure vessels and piping are mainly based on elastic analysis, elastic–plastic analysis and limit analysis. The stress classification method (SCM) [1–3] of ASME is one of the direct applications of elastic analysis. In design based on the SCM, the elastic stress is divided into primary, secondary and peak stresses. This method has been developed because of its relative simplicity and its general applicability. However, the evaluation method can be over-conservative and under-utilizes the load-carrying capacity of structures. Moreover, the SCM does not work well in some cases, especially for complex 3D structures.

Elastic-plastic analysis is based on the nonlinear constitutive relationship and can provide more precise solutions than elastic analysis. Recently many evaluation rules based on elastic-plastic analysis have been established. The elastic–plastic analysis method (EPAM) can evaluate the limit load relatively precisely, which generates a lower bound limit load at each step. A number of alternative methods, such as twice elastic slope method (TESM) [4–6], tangent intersection method (TIM) [5,7] etc. can be used to provide an engineering estimation of the limit load. Though these methods are widely used with the development of the computer, the calculation of limit loads is still difficult and expensive, especially for complex structures.

Mathematical programming methods [8–10] can also be used to determine the load-carrying capacity of structures. These do not follow the loading process and can overcome the difficulties of step-by-step elastic-plastic analysis. The lower and upper bound limit loads of a structure can be approached by mathematical programming processes based on the static and kinematic theorems of limit analysis. However, the complexity of mathematical programming restricts the application of this method.

Recently limit analysis based on elastic iterative procedures has become accepted widely in the engineering. Dhalla and Jones [11] established the reduced modulus method (RMM). The RMM simulates the effect of local high stress caused by the nonlinear material through an elastic analysis with the local elastic modulus reduced systematically. Marriott [12] considered the problem of stress classification using a variant of the RMM, and he proposed that the algorithm could be used to generate equilibrium stress fields suitable for application of the lower

* Corresponding author. Tel.: +86 10 62 77 3751; fax: +86 10 62 78 1824.
E-mail address: yhliu@mail.tsinghua.edu.cn (Y. Liu).

0308-0161/$ - see front matter © 2005 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijpvp.2005.06.005

bound limit load theorem. Seshadri and Fernando [13,14] established the GLOSS r-node method to calculate the plastic limit load by two linear elastic finite element evaluations, so the calculation is very cheap. Approaching the stress categorization problem in a similar way to Marriott but incorporating aspects of Seshadri's stiffness modification algorithm, Mackenzie and co-workers [15–18], Ponter and Carter [19] developed a new simple computational method called the elastic compensation method (ECM). The ECM uses conventional elastic finite element analysis to derive suitable stress and strain fields for the bounding theorems of classical plasticity. It requires only a few linear elastic finite element analyses of a structure. The analyses are carried out in such a way that the elastic modulus of each element in the model is adjusted to redistribute the stresses in order to simulate the formation of failure mechanisms. The redistributed elastic compensation stress and strain fields can then be applied to the lower and upper bound limit theorems. However, for some complex 3D structures, sometimes the ECM is not accurate [20], which will be demonstrated later in this paper. To overcome this problem, Ponter and Chen [21,22] developed an efficient method for the evaluation of limit and shakedown loads for complex structures based upon a linear matching method (LMM). The LMM is a generalization of the ECM and can obtain precise results even for complex structures. It is an upper bound method for the calculation of limit loads.

The present paper proposes modifications to the ECM to improve the computational precision for lower bound limit loads of complex structures. The modified elastic compensation method (MECM) is then used to perform limit analysis of nozzle-to-cylinder junctions. The calculated solutions are compared with other results from the elastic-plastic analysis method (EPAM) and the twice elastic slope method (TESM).

2. The elastic compensation method (ECM) [16]

According to limit analysis theorems, the ECM can obtain both upper bound and lower bound limit loads. Though the upper bound limit load given by the ECM is more accurate than the lower bound limit load [16], the lower bound limit load is safer. So the lower bound limit load needs to be obtained in some engineering fields such as structural design and safety assessment. In this paper, we present a modified lower bound method for the calculation of limit loads of complex structures, based on the ECM.

Based on an elastic-perfectly plastic material model, the ECM can simulate plastic failure mechanisms by adjusting the elastic modulus of each element systemically in a series of iterative elastic analyses. The elastic modulus of each element in a finite-element model is adjusted according to the ratio of the stress in the element to a nominal stress in order to redistribute the stress field away from highly stressed regions. A series of different statically admissible stress fields are obtained. The limit load can be determined from all these stress fields.

Initially, an elastic FEM analysis is performed under an arbitrary load p_n. The solution is considered as the first iteration for the series. Then the elastic modulus of each element is adjusted by the following equation

$$E_{i+1}^{e} = E_{i}^{e}\frac{\sigma_n}{\sigma_i^e} \tag{1}$$

where i is the iterative step, E_i^e is the current value of elastic modulus in the e-th element and E_{i+1}^e is the value for the next analysis in the series. σ_n is a nominal value of stress and σ_i^e is the maximum equivalent (von Mises yield criterion) stress associated with the element in the current solution. The value of σ_n is arbitrary and usually taken to be of the order of the nominal yield stress. In this paper, the nominal stress is always equal to $(\min_e(\sigma_i^e)+\max_e(\sigma_i^e)/2)$. Over a number of iterations, this procedure causes the stress in highly loaded elements to decrease while elements with initially low stress take more of the load. Thus, a series of equilibrium stress fields in which the highest value of stress is lower than that given in the initial analysis is defined. In the lower bound limit load procedure, these equilibrium stress fields are substituted into the lower bound theorem to establish lower bound limit loads for the structure. As the solution is linear, the maximum stress for solution i, $\max_e(\sigma_i^e)$, is proportional to the applied load P_n; thus the maximum load meeting the lower bound theorem maximum stress limit for solution i, p_{Li}, is obtained from proportionality

$$P_{\mathrm{Li}} = P_{\mathrm{n}}\frac{\sigma_Y}{\max_e(\sigma_i^e)} \tag{2}$$

where σ_y denotes the yield stress. The best estimate of lower bound limit load given by the ECM is the highest in the series of solutions:

$$P_{\mathrm{L}} = \max_i(P_{\mathrm{Li}}) \tag{3}$$

The limit load can be obtained efficiently by the above procedure. However, the ECM doesn't consider different plastic yield modes. According to our computational experiences, precise solutions can be obtained using the ECM if the structure yields globally at the limit state. However when the structure yields locally at the limit state, the computational error of the ECM is relatively great. Considering this situation, we developed a modified elastic compensation method (MECM) by introducing an adjustable factor f to improve the computational precision. This method is described in detail in Section 3.

3. A modified elastic compensation method (MECM)

In the ECM, the elastic moduli of all elements are modified during the iteration according to Eq. (1). However