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Wall thickness optimization of thick-walled spherical vessel using thermo-elasto-plastic concept

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Abstract

A study of thick-walled spherical vessels under steady-state radial temperature gradients using elasto-plastic analysis is reported. By considering a maximum plastic radius and using the *thermal autofrettage* method for the strengthening mechanism, the optimum wall thickness of the vessel for a given temperature gradient across the vessel is obtained. Finally, in the case of thermal loading on a vessel, the effect of convective heat transfer on the optimum thickness is considered, and a general formula for the optimum thickness and design graphs for several different cases are presented.

Keywords: Heat transfer; Elasto-plastic; Thick-walled vessels; Temperature gradient

1. Introduction

In general, the existence of any temperature gradient across the wall of a thick-walled vessel induces a thermal stress. Often, thermal stresses are greater than those generated by application of either internal and/or external pressure. From an economical point of view, the thermo-elasto-plastic method is used for design of such vessels. Detailed analyses of thermal stress in spherical and cylindrical vessels in the elastic range are given in [1–4]. In [5] the behaviour of thick-walled spherical and cylindrical vessels under thermal and mechanical stresses is considered. The exact solution for the stress distribution in a thick-walled sphere made of elastic-perfectly plastic material and under a steady state, radial temperature gradient is obtained in [6]. In the same paper an approximate solution with negligible elastic strain is also examined; the approximate and exact solutions yield the same results as the temperature gradient approaches infinity. The onset of yield in thick-walled spherical vessels for various combinations of temperature and pressure and various radius ratios is studied in [7]. Elasto-plastic thermal stresses in a spherical vessel under a temperature gradient across the wall thickness are studied in [8]. In all of these studies, for the thermal stress analysis, the temperature of the external surface of the vessel is held constant, at the same time, the temperature of its internal surface is increased and the resultant stresses are obtained [6–8]. In this paper, after reviewing some reported works in this field, by considering a maximum plastic radius and using the concept of thermal autofrettage for the strengthening mechanism, the analysis of thick-walled spheres with no convective heat transfer to the ambient is presented. Modeling and closed form solutions for stress distributions in the elastic part, due to combined pressure and temperature gradient, thermal loading and unloading and design curves are covered in Sections 2.1–2.5. However, in practice convective heat transfer occurs between the external surface of the vessel and the surroundings. This means that any changes in the temperature of the internal surface will change the temperature of the external surface. This change, in turn, is a function of the internal temperature, the size of the external surface, the mechanical properties of the vessel's material, and the properties of the fluid, which is in contact with the external surface of the vessel. In Section 3,

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the effect of convective heat transfer on the minimum thickness of the vessel, and design graphs for several different cases are presented.

2. Governing stress–strain relations

The following assumptions are made:

(1) The loading and geometry are symmetric, i.e. r and θ in spherical coordinates are principal planes and directions.
(2) Body forces are negligible.
(3) The vessel deformation is quasi-static, i.e. no stress waves are produced by applying a temperature gradient.
(4) The temperature of the inside surface of the vessel is greater than that of the outside surface, i.e. the direction of heat flow is outward.
(5) The temperature in the vessel is such that the effects of creep deformation are not considered.
(6) The vessel material is elastic-perfectly plastic.
(7) The yield stresses in tension and compression are the same, i.e. the Baushinger effect is neglected.
(8) The effects of stress and temperature on the modulus of elasticity, yield stress, coefficient of thermal expansion and the other material properties are negligible.
(9) The Tresca and von-Mises yield criteria are used.

2.1. The onset of yield under the coupled effect of a temperature Gradient and internal pressure

When a thick-walled sphere is under a severe temperature gradient, elastic and plastic zones are established. At the elasto-plastic interface, a radial stress is created that has a similar effect to an internal pressure on the elastic part of the vessel. In addition, because of the difference between the temperatures of the elasto-plastic interface and the outside radius, the elastic part of the vessel is under a temperature gradient. Therefore, the elastic part of the vessel is under the coupled effects of a temperature gradient and an internal pressure. In [7] this subject is studied in detail and the conditions for the onset of yield for different combinations of temperature and pressure are examined. The elastic stresses for combined loading of a thick-walled vessel are obtained from the following relations [7]:

$$\sigma_r/\beta = [m^3(1 - p/\beta) - mR^2(m^2 + m + 1) + R^3(p/\beta + m^2 + m)]/R^3(m^3 - 1) \quad (1)$$

$$\sigma_\theta/\beta = [-m^3(1 - p/\beta) - mR^2(m^2 + m + 1) + 2R^3(p/\beta + m^2 + m)]/2R^3(m^3 - 1) \quad (2)$$

In these relations $R=r/a$, $m=b/a$, and $\beta=\alpha E\Delta T/(1-\nu)$; a, b, ΔT, p, and α are the inside radius, outside radius, temperature difference between inside and outside radii, internal pressure, and heat expansion coefficient of the material, respectively. Denoting τ_r as the maximum shear stress at radius r, we have [7]:

$$\tau_r/\beta = (\sigma_\theta - \sigma_r)/2\beta = [3m^3(p/\beta - 1) + mR^2(m^2 + m + 1)]/4R^3(m^3 - 1) \quad (3)$$

First yield occurs at the radius at which τ_r reaches the value corresponding to the Tresca or von Mises yield criterion. For different combination of p and β and the radius ratio m, this radius can be at any position within the vessel wall. Fig. 1 shows these critical radii [7]. In regions I and IV, first yield is at the inside radius of the vessel. In region III, first

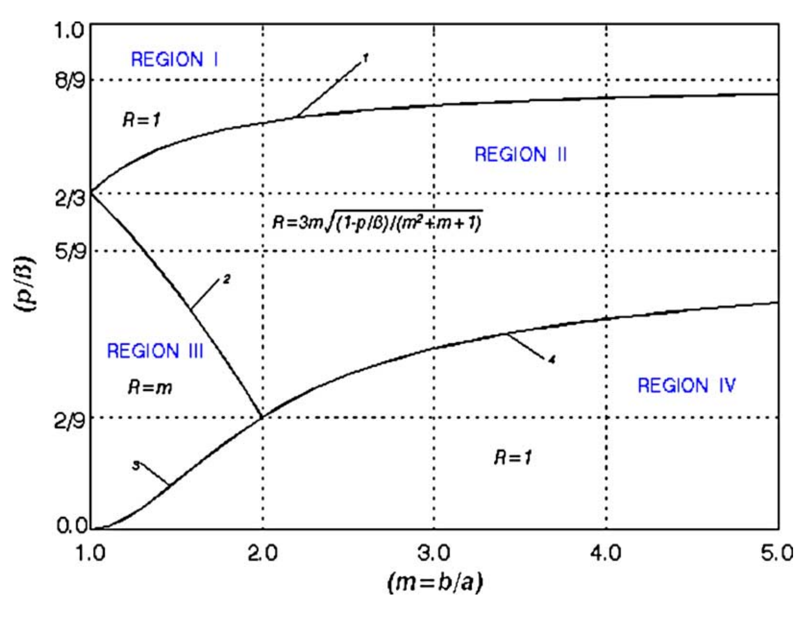

Fig. 1. Different regions for the onset of yield in a thick-walled sphere under combined internal pressure and radial temperature gradient.