Forensic Science International 198 (2010) 134–137

Contents lists available at ScienceDirect

Forensic Science International

journal homepage: www.elsevier.com/locate/forsciint

Two non-probabilistic methods for uncertainty analysis in accident reconstruction

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ARTICLE INFO

Article history:
Received 16 March 2009
Received in revised form 12 January 2010
Accepted 9 February 2010
Available online 6 March 2010

Keywords:
Accident reconstruction
Uncertainty analysis
Design of experiment
Interval analysis method
Convex models

ABSTRACT

There are many uncertain factors in traffic accidents, it is necessary to study the influence of these uncertain factors to improve the accuracy and confidence of accident reconstruction results. It is difficult to evaluate the uncertainty of calculation results if the expression of the reconstruction model is implicit and/or the distributions of the independent variables are unknown. Based on interval mathematics, convex models and design of experiment, two non-probabilistic methods were proposed. These two methods are efficient under conditions where existing uncertainty analysis methods can hardly work because the accident reconstruction model is implicit and/or the distributions of independent variables are unknown; and parameter sensitivity can be obtained from them too. An accident case is investigated by the methods proposed in the paper. Results show that the convex models method is the most conservative method, and the solution of interval analysis method is very close to the other methods. These two methods are a beneficial supplement to the existing uncertainty analysis methods.

1. Introduction

One of the basic purposes of traffic accident reconstruction is to define the pre-impact velocity and the collision location. On this basis, the driver's behavior is assessed, and the court takes its decision on whether the driver in the accident is proven guilty or not. However, the parameters in accident exhibit uncertainties due to the measurement errors, experience and other factors. Consequently, the accident reconstruction results, which often refer to pre-impact velocity and the collision location, are also uncertain.

The uncertainty of calculation results in accident reconstruction has been studied extensively. Many uncertainty analysis methods have been proposed, which can be divided into probabilistic methods and determined methods [1,2]. Probabilistic methods include Gauss Method [1,2], Methods with the Use of Stochastic Processes [1], Probabilistic Perturbation Method [3], Monte Carlo Method [4–6] and so on; determined methods include Total Differential Method [1,2], Upper and Lower Bound Method [1,2], Finite Difference Method [7,8] and so on. However, the probabilistic characteristic of the uncertain parameters is often scant, such that the distributed characteristics are difficult to obtain; probabilistic methods will hardly work under such condition. And most of these probabilistic methods and determined methods will hardly work if the expression of the accident reconstruction model is implicit. In these cases, alternative methods might be found useful.

During accident reconstruction, uncertainty analysis can be understood from the viewpoint of system reliability theory, which is to determine a response with respect to an input. So methods in system reliability will be helpful in uncertainty analysis of accident reconstruction. Interval mathematics and convex models are proved to be useful in system reliability analysis [9–11].

Design of Experiment (DOE) [12,13] is a method for organizing experiments in an efficient manner. It can not only be used for physical experimentation but also computational experimentation. Brach [14] used this method to analyze parametric sensitivity of planar impact mechanics. Design of Experiment can also be extended to uncertainty analysis.

In this paper, two approximate solution techniques are considered for accident reconstruction, which include uncertain-but-bounded parameters, are proposed based on interval mathematics, convex models and DOE. Finally, a numerical case is used to illustrate the applications of the presented methods.

2. Problem description

Accident reconstruction models vary with the accident nature, the selected methods and many other factors. But no matter which model is selected, it can be described by

$$Y = f(X), \quad X = (x_1, \ldots, x_s)^T \tag{1}$$

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doi:10.1016/j.forsciint.2010.02.006

where Y is the accident reconstruction results vector, which commonly represents the pre-impact velocity and the collision location; X is the independent variables vector, for example, dimension of the vehicle; s is the number of independent variables; f is an equation which characterizes the model.

The probabilistic distributions of independent variables are unknown and the expression of f is always implicit, the only known information is the interval of the uncertain parameters, namely

$$\underline{X} \le X \le \bar{X} \tag{2}$$

or the component form

$$\underline{x_i} \le x_i \le \overline{x_i}, \quad i = 1, 2, \ldots, s \tag{3}$$

in which $\underline{X} = (\underline{x_i})$ and $\bar{X} = (\overline{x_i})$ are, respectively, the lower bound vector and the upper bound vector of the uncertain parameters $X = (x_i)$. Under these conditions, evaluating the uncertainty of Y becomes a different issue. In this paper, two non-probabilistic methods will be proposed to solve the issue.

3. Interval analysis method based on DOE

In interval mathematics [15], the vector inequality constraint conditions (2) or (3) can be written as

$$X \in X^I = (x_i^I) = [\underline{X}, \bar{X}] \quad \text{or} \quad x_i^I = \left[\underline{x_i}, \overline{x_i}\right], \quad i = 1, 2, \ldots, s \tag{4}$$

where X^I is the s-dimension interval vector. Then the solution of Eq. (1) subject to Eq. (2) or (3) is a set, and this set is given by

$$\Gamma = \{Y : Y = f(X), X \in X^I\} \tag{5}$$

In general, the set Γ has a complicated region, but in interval mathematics, the upper and lower bounds on the set (5) will be sought as follows:

$$Y \in [\underline{Y}, \bar{Y}] \tag{6}$$

where $\bar{Y} = \max(\Gamma)$ and $\underline{Y} = \min(\Gamma)$, obviously, the maximum and minimum values are all global optimal solutions.

If the expression of f can be established, Eq. (6) can be obtained by means of the interval operations [15,16] from Eqs. (1) and (2) or (3)

$$Y \in [\underline{Y}, \bar{Y}] = f(X^I) \tag{7}$$

If the expression of f is implicit or very complicated, an approximate equation $g(X)$ will be found to replace f, and then, the interval of Y can be obtained according to interval operations and $g(X)$. After $g(X)$ is obtained, Eq. (7) can be written as

$$Y \in [\underline{Y}, \bar{Y}] = g(X^I) \tag{8}$$

The $g(X)$ is very important in this method. It should satisfy the following three conditions: 1, it should have an analytical expression; 2, the value of $|g(X) - f(X)|$ should satisfy the defined precision and the less the better; 3, it can be explained easily and has the main properties of the original model. There are many methods that can be used to find out $g(X)$, for example, Kriging method, base function method and so on.

Expanding $f(X)$ about $X = X_0$

$$f(X) = f(X_0) + \sum_{i=1}^{s} \frac{\partial f(X)}{\partial x_i}\bigg|_{x=x_0} (x_i - x_{i0}) + \frac{1}{2!}\sum_{i=1}^{s} \frac{\partial^2 f(X)}{\partial x_i^2}\bigg|_{x=x_0} (x_i - x_{i0})^2 + \cdots \tag{9}$$

where X_0 is the nominal value of X; X_i is the ith independent variable, X_{i0} is the corresponding nominal value. Uncertainty of each independent variable is frequently small, so that second order terms and higher order terms can be ignored. So

$$f(X) \approx f(X_0) + \sum_{i=1}^{s} \frac{\partial f(X)}{\partial x_i}\bigg|_{x=x_0} (x_i - x_{i0}) = a_0 + BX, \quad X = (x_1, \ldots, x_s)^T \tag{10}$$

where

$$a_0 = f(X_0) - BX_0, \quad B = \left(\frac{\partial f(X)}{\partial x_1}, \ldots, \frac{\partial f(X)}{\partial x_s}\right)\Big|_{X=X_0}.$$

Therefore as a general rule, the expression of $g(X)$ can be chosen as

$$Y = g(X) = d_0 + DX, \quad X = (x_1, \ldots, x_s)^T, \quad D = (d_1, d_2 \ldots d_s) \tag{11}$$

where d_0 and D are the undetermined coefficient. If Eq. (11) cannot satisfy the listed conditions, then the higher order terms in Eq. (9) should be reserved, and Eq. (11) needs to be changed accordingly.

After d_0 and D in Eq. (11) are obtained, Eq. (8) can be given as

$$Y \in [\underline{Y}, \bar{Y}] = d_0 + \sum_{i=1}^{s} d_i [\underline{x_i}, \overline{x_i}] \tag{12}$$

On the basis of the above, steps of the interval analysis method, based on DOE, can be given:

1. *To choose independent variables.* There are many uncertain independent variables in one problem, and each variable may influence results. To consider the influence of all variables would be both difficult and unnecessary. According to the character of the accident reconstruction model and experience, the appropriate independent variables should be chosen, then the upper and lower bounds of each independent variable should be obtained.
2. *To choose an appropriate DOE method.* There are also many DOE methods. As one of the many choices, uniform design is a main method of fractional factorial design, it is an important DOE method in computer simulation, and it is also a robust DOE method [13]. Thereby, uniform design is chosen in this paper.
3. *To conduct experiment.* After the level of each independent variable and the uniform design table are fixed, according to the use-table of the uniform design table, arrange the independent variable into the uniform design table, and then, the experiment table can be obtained after level translation. Finally, conduct experimentation according to the experiment table strictly and record the results.
4. *To obtain the regression equation* $g(X)$. Select the expression of $g(X)$ according to the characterization of the problem, the selected model and experience. Use regression analysis method to analyze the experiment results, and then $g(X)$ can be obtained.
5. *To analyze the uncertainty of accident reconstruction results.* The upper and lower bounds of accident reconstruction results can be obtained by Eq. (12).

4. Convex models based on DOE

Based on the interval transformation, Eq. (4) can be put into the more useful form

$$x_i^I = x_i^c + [-r_i, r_i], \quad i = 1, 2, \ldots, s \tag{13}$$

where x_i^c and r_i denote the middle value and the maximum error of x_i^I, respectively. It follows that

$$x_i^c = \frac{\underline{x_i} + \overline{x_i}}{2}, \quad r_i = \frac{\underline{x_i} - \overline{x_i}}{2}, \quad i = 1, 2, \ldots, s \tag{14}$$

As the interval analysis method is based on DOE, convex models, based on DOE are also needed to find an approximate