Journal of Materials Processing Technology 160 (2005) 370–373

Journal of Materials Processing Technology

www.elsevier.com/locate/jmatprotec

On the thinning variation of a superplastically formed titanium alloy spherical domes

J.J.V. Jeyasingh[a], B. Nageswara Rao[b,*]

[a] *Computer Aided Design Facility, Mechanical Engineering Entity, Vikram Sarabhai Space Centre, Thiruvananthapuram 695022, India*
[b] *Structural Analysis and Testing Group, Aeronautics Entity, Vikram Sarabhai Space Centre, Thiruvananthapuram 695022, India*

Received 12 September 2003; received in revised form 24 June 2004; accepted 24 June 2004

Abstract

The gas pressure bulging of metal sheets has become an important forming method. As the bulging process progresses, significant thinning in the sheet material becomes obvious. A prior knowledge about non-uniform thinning in the product after forming helps the designer in the selection of initial blank thickness. This paper presents a simple analytical procedure for obtaining the thinning variation of a superplastically formed Ti alloy spherical dome. The procedure is validated with the existing measured data.

Keywords: Superplastic deformation; Bulge forming; Spherical domes

Superplastic forming has become a promising processing technique in manufacturing industry. Several models for superplastic bulge forming have been established. Jovane [1] has simulated hemispherical shell without considering the non-uniformity of thickness from apex to periphery. Ragab [2] has assumed a balanced bi-axial stress state throughout the dome profiles, which is only correct on the apex of the dome where the tangential stress is equal to the circumferential stress. Cornfield and Johnson [3] have taken into account the effect of strain rate sensitivity index (m) on thickness with the same balanced bi-axial assumption. At the pole of the hemispherical dome, the orthogonal stresses are equal, and the stress state is that of equi-biaxial tensile. At the edge of the dome, there is a constraint around the periphery, leading to a plane-strain stress state. Therefore, the stress gradient in a forming dome causes a more rapid thinning rate at the pole, and it may be expected that the thinning difference will accelerate with time, leading to a thickness gradient in the formed dome. A number of analytical developments have been made to predict the thinning variation of the superplastically formed spherical domes [4–6]. Cheng [6] suggested an empirical relation based on the measured thickness values:

$$s = s_p + (s_e - s_p)\left(\frac{l}{L}\right)^2 \tag{1}$$

where s_p and s_e are the measured thicknesses at the pole and edge of the dome. 'l' is the arc length measured from the pole towards the edge of the dome where the thickness (s) is measured. L is the arc length of the dome.

A simple analytical procedure for obtaining the thinning variation of a superplastically formed spherical dome is presented, which is validated by comparing the results with those measured on a superplastically formed Ti alloy spherical dome. In the present analysis, the following assumptions are made: the material obeys von Mises' effective stress and strain criteria; the volume of the deforming metal remains constant at any instant of forming; and the flow stress in the thickness direction is ignored for the deformation in a thin membrane.

von Mises' effective stress (σ_{eff}) and effective strain (ε_{eff}) are defined by,

$$\sigma_{eff} = \frac{1}{\sqrt{2}}[(\sigma_\theta - \sigma_m)^2 + (\sigma_m - \sigma_s)^2 + (\sigma_s - \sigma_\theta)^2]^{1/2} \tag{2}$$

* Corresponding author. Tel.: +91 471 2565640; fax: +91 471 2704134.
E-mail address: bnrao52@rediffmail.com (B.N. Rao).

0924-0136/$ – see front matter © 2004 Elsevier B.V. All rights reserved.
doi:10.1016/j.jmatprotec.2004.06.024

$$\varepsilon_{\text{eff}} = \frac{\sqrt{2}}{3}[(\varepsilon_\theta - \varepsilon_m)^2 + (\varepsilon_m - \varepsilon_s)^2 + (\varepsilon_s - \varepsilon_\theta)^2]^{1/2} \quad (3)$$

where σ_θ, σ_m, and σ_s are the hoop, meridional and thickness stresses. ε_θ, ε_m, and ε_s are respectively, the corresponding strains.

The relation between principal stresses and strains is:

$$\varepsilon_\theta = \frac{1}{E_p}\left[\sigma_\theta - \frac{1}{2}(\sigma_m + \sigma_s)\right] \quad (4)$$

$$\varepsilon_m = \frac{1}{E_p}\left[\sigma_m - \frac{1}{2}(\sigma_\theta + \sigma_s)\right] \quad (5)$$

$$\varepsilon_s = \frac{1}{E_p}\left[\sigma_s - \frac{1}{2}(\sigma_\theta + \sigma_m)\right] \quad (6)$$

where E_p is the plastic modulus.

The volume of deforming metal remains constant which implies that

$$\varepsilon_\theta + \varepsilon_m + \varepsilon_s = 0 \quad (7)$$

The flow stress in the thickness direction is ignored, i.e.,

$$\sigma_s = 0 \quad (8)$$

Using Eq. (8) in Eqs. (4) and (5), and eliminating E_p, one can obtain,

$$\sigma_m = \left(\frac{2\kappa + 1}{\kappa + 2}\right)\sigma_\theta \quad (9)$$

where

$$\kappa = \frac{\varepsilon_m}{\varepsilon_\theta} \quad (10)$$

Using Eqs. (8) and (10) in Eq. (2), one obtains

$$\sigma_{\text{eff}} = \beta\sigma_\theta \quad (11)$$

where

$$\beta = \frac{\sqrt{3(1 + \kappa + \kappa^2)}}{\kappa + 2} \quad (12)$$

Using Eqs. (7) and (10) in Eq. (3), one can write

$$\varepsilon_{\text{eff}} = -\gamma\varepsilon_s \quad (13)$$

where

$$\gamma = \frac{2}{\sqrt{3}}\frac{\sqrt{1 + \kappa + \kappa^2}}{\kappa + 1} \quad (14)$$

Taking into consideration the symmetry of a preform, let us consider bulging of one circular membrane with initial die radius, a. The initial sheet thickness, s_0, is assumed to be small in comparison with the die radius, a (i.e., $s_0 \ll a$). The shape of the forming dome is supposed to be a part of the sphere with the current radius, ρ. The material is assumed to be isotropic and incompressible, while flow stress depends on strain rate and temperature.

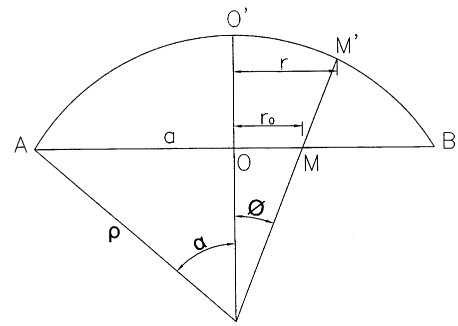

Fig. 1. Schematic representation of bulge forming of select.

Let M be some point on the membrane, which at the initial moment of time ($t = 0$) belongs to the diameter AB (see Fig. 1). The envelope is clamped around its periphery and r_0 is the distance between point M and the centre of membrane O. At some current moment of time $t > 0$, point M goes to M′, while point O goes to point O′.

The hoop, meridional and thickness strains at M′ are,

$$\varepsilon_\theta = \ell n\left(\frac{\rho\alpha}{a}\right) \quad (15)$$

$$\varepsilon_m = \ell n\left(\frac{2\pi r}{2\pi r_0}\right) = \ell n\left(\frac{\rho\alpha}{a}\frac{\sin\phi}{\phi}\right) \quad (16)$$

$$\varepsilon_s = \ell n\left(\frac{s_\phi}{s_0}\right) \quad (17)$$

where s_ϕ is the thickness at M′. α is the half angle subtended by the dome surface at its centre of curvature and ϕ is the angle between the symmetry axis and the dome radius, drawing to the point under consideration.

$$r = \rho\sin\phi, \quad (18)$$

is the distance between M′ and the symmetry axis.

From the proportionality,

$$\frac{\rho\phi}{r_0} = \frac{\rho\alpha}{a} \quad (19)$$

The angle, α can be obtained from

$$\sin\alpha = \frac{a}{\rho} \quad (20)$$

Within the limited range of strain rates, the flow stress data of the material from uniaxial tests can be represented by the power-law relation:

$$\sigma = K(\dot{\varepsilon})^m \quad (21)$$

where K and m are material constants. 'm' in Eq. (21) is known as the strain rate sensitivity index of the material. The effective stress and strain obey the power-law relation (21).

Using Eqs. (11)–(17) in the power-law (21), one can obtain

$$\beta\sigma_\theta = K\left\{\frac{\gamma}{t}\ell n\left(\frac{s_0}{s_\phi}\right)\right\}^m \quad (22)$$