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Duality in comparative statics in rental housing markets with indivisibilities

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Abstract

We present certain duality results on comparative statics on competitive rent vectors in the rental housing market model. In the model, apartments as indivisible goods are classified into a finite number of categories, and are traded for one composite commodity. Our concern is about certain general properties of the behavior of rents with parameter changes. In particular, the rent changes are intimately related to the boundary income changes of the categories of apartments. Both changes are endogenously determined in equilibrium. We will show that these changes exhibit nice dual structures. We will also apply our model and comparative statics to a rental housing market in the Tokyo metropolitan area.

JEL classification: R31; R30; D58

Keywords: Rental housing; Apartment; Indivisible goods; Competitive equilibrium; Rent equation; Comparative statics; Duality

1. Introduction

This paper aims to present certain duality results on comparative statics on competitive rent vectors in the rental housing market model. In the model, apartments as indivisible goods are classified into a finite number of categories by some attributes such as sizes, commuting times and housing regions, and they are traded for one composite commodity. By comparative statics, we mean a study of the behavior of rents when some parameters of the market change. Our concern

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is not a particular form of "cause–effect" comparative statics, but it is about certain general properties of the behavior of rents with parameter changes. We focus on the boundary income, i.e., the lowest income, in each category. The changes in rent differences are intimately related to the changes in boundary income differences. Both changes are endogenously determined in equilibrium. We will show that these changes exhibit nice dual structures.

This introduction will first give brief explanations of the relation of our housing model to some urban economics literatures as well as to the literature of markets with indivisible goods, and then will explain our comparative statics.

Our model is a partial equilibrium model in that all goods other than housing are treated as one composite commodity. But it admits commodity differentiations in apartments as well as income effects on households' behavior. Our model is a variant of the bid–offer model in the standard urban economics literature from Alonso [1] and Muth [18] (see Fujita [7] for a recent textbook), where housing sizes are treated as continuous variables. A salient difference of our model from this literature is that each housing unit is indivisible, and also, the number of apartment units is finite.

In another literature in urban economics such as Sweeney [22], Braid [5] and van Lierop [24], apartments are treated as indivisible. However, the housing quality is expressed as a continuous variable. The mathematical methodology used in these urban economics literatures is dominantly calculus and analysis. Our approach differs from those literatures in that our housing model is of finite nature, except the composite commodity. This requires a different mathematical method, which is of combinatorics nature. We show that this finite nature is able to capture the housing market structure well.

The mathematical model of this paper also belongs to the other tradition from Böhm-Bawerk [25], Neumann–Morgenstern [26] and Shapley–Shubik [21] (see Laan et al. [16] and its references for recent papers). Nevertheless, this tradition has had a theoretical focus on the existence of competitive equilibria and the nonemptiness of the core, etc. Only Kaneko [15] and Gerber [9] tried to apply this approach to urban housing market problems. The present paper is a further development of [15].

Our approach has both advantages and disadvantages to the standard bid–offer approach. It is a disadvantage, caused by the indivisibility assumption, that the finite number of housing sizes are already fixed, while a household can choose freely a size or quality in the standard approach. This disadvantage restricts our scope to housing markets where houses are dwellings, i.e., they are already built and their sizes are fixed. With the cost of this disadvantage, our restriction can be regarded as advantageous in a quite few respects over the standard approach. For example, it will be clear by giving certain numerical examples that our model is flexible enough to capture rental housing markets well. The main theme of this paper, i.e., comparative statics, is also an advantage.

We should emphasize that our approach is short-run in the sense that we treat housing units as already produced and provided to the market. It may be found in the numerical examples given in Sections 3 and 7 that our approach is even regarded as snapshot in that the housing market at a point of time is studied. This is in contrast with the long-run approach such as in Braid [6].[1]

Let us give a brief description of our housing market model. Our model has various differentiated apartments and one composite commodity. Section 2 will start with a general formulation, while referring to some preceding works on the existence of a competitive equilibrium. There the

[1] Cf., also Sweeney [23], Ohls [20] and Arnott–Braid [2].