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A one-dimensional theory of strain-gradient plasticity: Formulation, analysis, numerical results

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Abstract

This study develops a one-dimensional theory of strain-gradient plasticity based on: (i) a system of microstresses consistent with a microforce balance; (ii) a mechanical version of the second law that includes, via microstresses, work performed during viscoplastic flow; (iii) a constitutive theory that allows

- the free-energy to depend on the gradient of the plastic strain, and
- the microstresses to depend on the gradient of the plastic strain-rate.

The constitutive equations, whose rate-dependence is of power-law form, are endowed with energetic and dissipative gradient length-scales L and l, respectively, and allow for a gradient-dependent generalization of standard internal-variable hardening. The microforce balance when augmented by the constitutive relations for the microstresses results in a nonlocal flow rule in the form of a partial differential equation for the plastic strain. Typical macroscopic boundary conditions are supplemented by nonstandard microscopic boundary conditions associated with flow, and properties of the resulting boundary-value problem are studied both

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analytically and numerically. The resulting solutions are shown to exhibit three distinct physical phenomena:

(i) standard (isotropic) *internal-variable hardening*;
(ii) *energetic hardening*, with concomitant back stress, associated with plastic-strain gradients and resulting in boundary layer effects;
(iii) *dissipative strengthening* associated with plastic strain-rate gradients and resulting in a size-dependent increase in yield strength.

Keywords: Constitutive behavior; Viscoplasticity; Strain-gradient plasticity; Finite elements

1. Introduction

A number of experimental results including those from nano/micro-indentation, torsion of micron-dimensioned wires, and bending of micron-dimensioned thin-films, all show that in the approximate size-range 500 nm–50μm, the strength of metallic components undergoing inhomogeneous plastic flow is inherently size-dependent, with *smaller being stronger*.[1] Because conventional plasticity theories do not contain intrinsic material length-scales, such theories cannot describe size-dependent phenomena, a drawback that has led to the development of theories that capture such phenomena via dependencies on plastic-strain gradients. There is a large and growing literature on theories of this type; references *relevant to our work* may be found in Gurtin and Anand (2004a).

In forthcoming papers Gurtin and Anand (2004a) (small deformations) and Gurtin and Anand (2004b) (finite deformations) develop strain-gradient theories for isotropic plastic materials based on a general framework for gradient plasticity developed by Gurtin.[2] The central theme underlying Gurtin's work is an accounting for the power expended by microstresses in consort with temporal changes in plastic strain and plastic-strain gradient. This theme leads, in a natural manner, to a microforce balance for the microstresses that, when combined with thermodynamically consistent constitutive equations, forms the flow rule of the theory.

Because of the complicated nature of the partial differential equations that represent the flow rules proposed by Gurtin and Anand (2004a, b), it is not at all clear what general physical phenomena are characterized by the resulting theory. For that reason, we here develop a corresponding *one-dimensional* (1D) theory for a body in the form of a strip of finite width ($0 \leqslant y \leqslant h$) undergoing simple shear with shear stress τ. Under a simplified set of constitutive relations, that hopefully extract the essence of the general theory, our 1D theory leads to a *nonlocal* flow rule in the form

[1]Cf., e.g., Stelmashenko et al. (1993), Ma and Clarke (1995), Fleck et al. (1994), Stolken and Evans (1998), Hutchinson (2000).

[2]Gurtin (2000, 2002) (single crystals), Gurtin (2003, 2004) (isotropic plasticity). See also Gudmundson (2004), who (more or less) uses this framework to develop a theory to which Gurtin and Anand (2004a) is close in spirit.