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Mechanism-based strain gradient crystal plasticity—I. Theory

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Abstract

We have been developing the theory of mechanism-based strain gradient plasticity (MSG) to model size-dependent plastic deformation at micron and submicron length scales. The core idea has been to incorporate the concept of geometrically necessary dislocations into the continuum plastic constitutive laws via the Taylor hardening relation. Here we extend this effort to develop a mechanism-based strain gradient theory of crystal plasticity. In this theory, an effective density of geometrically necessary dislocations for a specific slip plane is introduced via a continuum analog of the Peach–Koehler force in dislocation theory and is incorporated into the plastic constitutive laws via the Taylor relation.

Keywords: A. Dislocations; Strengthening mechanisms; B. Constitutive behavior; Principles; Elastic–plastic material; Strain gradient plasticity

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1. Introduction

Strain gradient plasticity theories (e.g., Aifantis, 1987; Fleck et al., 1994; Nix and Gao, 1998; Gao et al., 1999, Huang et al., 2000a) have been developed to model size effects of plasticity at micron and submicron length scales. Nix and Gao (1998) analyzed micro-indentation experiments and found that the measured indentation hardness strongly suggests a law of linear dependence between the square of plastic flow stress and strain gradient. Interestingly, while such linear dependence is predicted by the classical Taylor relation $\tau = \alpha\mu b\sqrt{\rho}$ between flow stress τ and dislocation density ρ, where μ denotes the shear modulus, b the Burgers vector length and α an empirical coefficient usually taken to be 0.2–0.5, many existing theories of strain gradient plasticity failed to explain this behavior. This study has subsequently led to the development of mechanism-based strain gradient (MSG) plasticity by Gao et al. (1999) and Huang et al. (2000a). The core idea of MSG has been to incorporate the concept of geometrically necessary dislocations into the continuum plastic formulation via the Taylor (1934) relation

$$\tau = \alpha\mu b\sqrt{\rho_{\mathrm{S}} + \rho_{\mathrm{G}}},$$

where the dislocation density is divided into the density of statistically stored dislocations ρ_{S} and that of geometrically necessary dislocations ρ_{G}. The MSG theory has been used to model size-dependent deformation in a number of applications (Huang et al., 2000a, b; Shi et al., 2000; Guo et al., 2001; Hwang et al., 2002, 2003; Huang et al., 2004). Besides the model originally envisaged by Taylor (1934), a linear relation between τ and $\sqrt{\rho}$ is also suggested by other hardening mechanisms and confirmed by experimental observations over several orders of magnitude in variations of dislocation density in many materials (e.g., Courtney, 1990). Mott (1952) considered dislocations nucleated sequentially from a fixed source. As the leading dislocation comes to rest, the resulting dislocation pile-up give rise to stresses that may prevent further operation of the source. This concept yields essentially the same relation as that of Taylor (Wood, 1971).

Most existing theories of strain gradient plasticity have been formulated for isotropic materials. With decreasing size scale, the inherent anisotropy of crystal lattices is expected to become increasingly important and crystal plasticity offers a better description of lattice anisotropy at the size scale of a few micrometers at which significant size effects are observed. Ma and Clarke (1995) measured different indentation hardness of silver crystals along (1 0 0) and (1 1 0) planes. While isotropic strain gradient formulations are capable of qualitatively explaining the size dependence of indentation hardness (Nix and Gao, 1998), different material parameters are often needed to fit experimental data in different crystal orientations.

An implicit assumption of conventional crystal plasticity is that the lattice remains undistorted during plastic deformation. Such theories have no internal length scales and are normally not capable of predicting size effects. Strain gradient formulations of crystal plasticity have been previously proposed by Smyshlyaev and Fleck (1996) and by Shu and Fleck (1998) within a higher order continuum framework. Acharya and Bassani (2000) incorporated strain gradients into the work hardening moduli via