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Mechanism-based strain gradient crystal plasticity—II. Analysis

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Abstract

In part I of this series (Mechanism-based strain gradient crystal plasticity—I. Theory. J. Mech. Phys. Sol. (2005), accepted for publication), we have proposed a theory of mechanism-based strain gradient crystal plasticity (MSG-CP) to model the effect of inherent anisotropy of a crystal lattice on size-dependent non-uniform plastic deformation at micron and submicron length scales. In the present paper, several example problems are investigated to show how crystal anisotropy is reflected by the MSG-CP theory.

Keywords: Dislocations; Strengthening mechanisms; Constitutive behavior; Principles; Elastic–plastic material; Strain gradient plasticity; Crystal plasticity; GNDS

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1. MSG-CP theory

The present paper is aimed to present some analysis of the theory of mechanism-based strain gradient crystal plasticity (MSG-CP) proposed in part I of this series (Han et al., 2005) to model the effect of crystal anisotropy on size-dependent non-uniform plastic deformation at micron and submicron length scales. For the convenience of the reader, the basic formulation of MSG-CP is briefly summarized below. We restrict our attention to small deformation and ignore elastic strain. In this case, the displacement gradient can be written as $\nabla \mathbf{u} = \mathbf{H}_\star + \mathbf{H}_\mathrm{p}$ where $\mathbf{H}_\mathrm{p}$ denotes plastic distortion and $\mathbf{H}_\star$ reflects lattice rotation (Cermelli and Gurtin, 2001). The constitutive relations of conventional crystal plasticity are summarized in Table 1. The evolution of slip resistance is often characterized by linear strain hardening with coefficients $h^{\alpha\beta} = [q + (1-q)\delta^{\alpha\beta}]h^\beta$, as in Eq. (6). For the analysis in this paper, it will be assumed that the deformation remains in the range of easy glide so that the effects of cross and latent hardening are negligible. We assume $q = 1$ and constant self-hardening, with $h^\beta = c_h = constant$. These assumptions lead to a simple expression for the slip resistance evolution

$$\dot{g}^\alpha = c_h \sum_\beta |\dot{\gamma}^\beta| = c_h \dot{\gamma}, \tag{1}$$

where $\gamma = \int_t \sum_\alpha |\dot{\gamma}^\alpha| \, \mathrm{d}t$ is the accumulated plastic strain.

An implicit assumption of conventional crystal plasticity is that the plastic deformation leaves the lattice essentially undistorted. Such theories are in general not capable of describing size effects observed at small length scales. In contrast, plastic strain gradients can be used to represent lattice distortions due to geometrically necessary dislocations which give rise to extra hardening in the material. In the mechanism-based strain gradient (MSG) plasticity (Nix and Gao, 1998; Gao et al., 1999; Huang et al., 2000, 2004), the total dislocation density ρ is decomposed into the density of statistically stored dislocations ρ_S which is related to plastic strain and that of geometrically necessary dislocations ρ_G which is related to the gradients in plastic strain. A law for strain gradient plasticity is then established via the Taylor

Table 1
Summary of constitutive relations of rate-dependent crystal plasticity at small deformation

Displacement gradient:	$\nabla \mathbf{u} = \mathbf{H}_* + \mathbf{H}_\mathrm{p}$	(2)
Plastic distortion:	$\mathbf{H}_\mathrm{p} = \sum_\alpha \gamma^\alpha \mathbf{s}^\alpha \otimes \mathbf{m}^\alpha$	(3)
Plastic strain:	$\boldsymbol{\varepsilon}_p = \sum_\alpha \gamma^\alpha (\mathbf{s}^\alpha \otimes \mathbf{m}^\alpha)_\mathrm{sym}$	(4)
Resolved shear stress:	$\tau^\alpha = \boldsymbol{\tau} \cdot (\mathbf{s}^\alpha \otimes \mathbf{m}^\alpha)$	(5)
Rate of slip resistance:	$\dot{g}^\alpha = \sum_\beta h^{\alpha\beta} \lvert\dot{\gamma}^\beta\rvert$	(6)
Power-type law:	$\dot{\gamma}^\alpha = \dot{\gamma}_0 \left\lvert \frac{\tau^\alpha}{g^\alpha} \right\rvert^n \mathrm{sign}(\tau^\alpha)$	(7)