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Composite and filament models for the mechanical behaviour of elastomeric materials

Dmitri Miroshnychenko[a,*], W.A. Green[a], D.M. Turner[b]

[a]*Department of Mathematical Sciences, University of Loughborough, Loughborough, Leicestershire LE11 3TU, UK*
[b]*Consultant, Swithuns Gate, Ostlings Lane, Bathford, Bath BA1 7RW, UK*

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Abstract

In this work we present a composite model, which combines the approach of Poisson's function with the filament theory and requires three material parameters. We also suggest the form for a strain-energy function that approximates the constitutive equations of the composite model. Furthermore, a simple asymptotic analysis allows us to reduce the number of material constants to only two, thus, forming a new filament model. The predictive capability of the two models to reproduce the mechanical behaviour of elastomeric materials in deformation experiments is evaluated against the extensive data of Kawabata et al. (Macromolecules 14 (1981) 154). The models give excellent agreement in not only uniaxial and equibiaxial but also non-equibiaxial extension. Although being rather more simplistic in comparison with some successful network models involving non-Gaussian chain statistics, the two models conform much more closely to the classical experimental data of Treloar (Trans. Faraday Soc. 40 (1944) 59).

Keywords: Deformation; Constitutive behaviour; Polymeric material; Polymer network; Finite strain

*Corresponding author. Present address: Department of Mathematics, University of Glasgow, 15 University Gardens, Glasgow G12 8QW, UK. Tel.: +44 141 330 4746; fax: +44 141 330 4111.
E-mail address: dm@maths.gla.ac.uk (D. Miroshnychenko).

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1. Introduction

The mechanical behaviour of elastomeric materials under static load is highly nonlinear elastic at moderate to large stretches. Although there have been developed some very successful network models involving chain statistics, which reproduce a stress–stretch curve in uniaxial experiments very well, accurate subsequent prediction of the behaviour in biaxial deformation still remains a barrier for many of them.

The ability to perform these predictions with a greater precision can in turn lead to the identification of more thorough design criteria for industrial applications of polymeric materials. Hence, more reliable models are needed which would be of particular interest to product designers, plastics processors, and polymer suppliers as well as to theoreticians in nonlinear elasticity.

Existing models provide constitutive equations for isotropic, incompressible and highly nonlinear elastic materials in the forms obtained by constructing a strain-energy function. This is usually derived according to either phenomenological (empirical evidence and mathematical developments) or physical (chain statistics) considerations.

The phenomenological approach leads to the strain energy as a function either of invariants of the stretch tensor (Mooney, 1940; Rivlin, 1948; Rivlin and Saunders, 1951; Gent and Thomas, 1958; Hart-Smith, 1966) or of the principal stretch ratios (Valanis and Landel, 1967; Ogden, 1972). For more details on the subject of phenomenological modelling in nonlinear elasticity refer, for instance, to Ogden (1997).

Physical considerations lead to structural models, which relate the macroscopic behaviour of elastomeric materials to deformations in their polymer networks. In those models a polymer network consists of long flexible chain molecules randomly oriented and joined together by chemical cross-links. The strain energy (or the work of deformation) can be calculated from a reduction of entropy in the network upon stretching the chains. Further details on structural modelling can be found in the classical book by Treloar (1975).

Models for a single chain (Kuhn and Grün, 1942) and for networks of three (James and Guth, 1943; Wang and Guth, 1952), four (Flory and Rehner, 1943; Flory, 1944; Treloar, 1946), eight (Arruda and Boyce, 1993), and a large assembly (Treloar and Riding, 1979) of chains have been developed using Gaussian or primarily non-Gaussian (Langevin) statistics. Some other statistical models (Flory and Erman, 1982) consider phantom deformation and fluctuation of junctions in polymer networks as well, and those have been discussed in Mark and Erman (1988).

Comparisons (Arruda and Boyce, 1993; Wu and van der Giessen, 1993) of the predictive capability of these structural and phenomenological models suggest that the most effective, successful and simple ones are the eight-chain model and the Ogden model. The latter captures the highly nonlinear elastic behaviour of rubber-like materials in shear, uniaxial and biaxial extension (Treloar, 1944) more accurately than the former, however, the Ogden model requires six parameters in comparison with only two for the eight-chain model.