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The effect of long-range forces on the dynamics of a bar

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Abstract

The one-dimensional dynamic response of an infinite bar composed of a linear "microelastic material" is examined. The principal physical characteristic of this constitutive model is that it accounts for the effects of long-range forces. The general theory that describes our setting, including the accompanying equation of motion, was developed independently by Kunin (Elastic Media with Microstructure I, 1982), Rogula (Nonlocal Theory of Material Media, 1982) and Silling (J. Mech. Phys. Solids 48 (2000) 175), and is called the peridynamic theory. The general initial-value problem is solved and the motion is found to be dispersive as a consequence of the long-range forces. The result converges, in the limit of short-range forces, to the classical result for a linearly elastic medium. Explicit solutions in elementary form are given in a broad class of special cases. The most striking observations arise in the Riemann-like problem corresponding to a constant initial displacement field and a piecewise constant initial velocity field. Even though, initially, the displacement field is continuous, it involves a jump discontinuity for all later times, the Lagrangian location of which remains stationary. For some materials the magnitude of the discontinuity-jump oscillates about an average value, while for others it

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grows monotonically, presumably fracturing the material when it exceeds some critical level.

Keywords: Displacement jump; Fracture; Long-range; Microelastic; Nonlocal; Peridynamic

1. Introduction

In this paper we examine the one-dimensional dynamic response of an infinite bar composed of a linear *microelastic material*. The principal physical characteristic of such material model is that it accounts for the effects of long-range forces. The analysis is carried out in the continuum setting of the "peridynamic theory" (Silling, 2000).

The peridynamic theory is effectively an integral-type nonlocal model, which, in contrast to other such models, only involves the displacement field, *not* its gradient (Section 2). This leads to a theory that formally appears to be a continuum version of molecular dynamics. However this similarity is misleading since the peridynamic theory is meant to apply at length-scales *between* those of classical continuum mechanics and molecular dynamics. In particular, the pairwise force function in this continuum theory is not meant to coincide with the interatomic force function in a molecular dynamic model of the same material. An attractive feature of peridynamic theory is the computational advantage resulting from the absence of spatial gradients, especially in settings that involve singularities. For a detailed description of a variety of nonlocal continuum theories see, for example, Kunin (1982) and Rogula (1982); a discussion of the relation between nonlocal theories and atomistic models can be found in, e.g., Chen et al. (2004).

In a general initial-value problem, the bar is subjected to a prescribed time-dependent loading field and the motion commences from some given initial state.[1] This problem is solved using Fourier-transform methods in Section 3 and the result is expressed in terms of a Green's function. In Section 5 we use this result to obtain explicit solutions, in elementary form, in the special case where (i) the initial data is spatially periodic and (ii) the prescribed loading is separable into a harmonic time component and a periodic spatial component. The motion is dispersive as a result of the long-range forces.

The general result converges in the limit of short-range forces to the classical result for a linearly elastic medium. In Section 4 we derive the first-order correction to linear elasticity for small values of the length-scale parameter that characterizes nonlocality.

Perhaps the most striking observations are those made in Section 6 where we consider a Riemann-like problem. Here, the body is subjected to a zero initial displacement field and a piecewise constant initial velocity field. There is no external loading. Even though, initially, the displacement field is continuous, it *involves a*

[1]The corresponding static problem has been studied previously in Silling et al. (2003).