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Comprehensive bounded asymptotic solutions for incomplete contacts in partial slip

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Abstract

Solutions for the traction distributions and corresponding sub-surface state of stress adjacent to the edge of an incomplete contact suffering partial slip are found. The effects of frictional shakedown and a *synchronously varying* in-plane tension on the solution are found in closed form. The value of the asymptote, and its characterisation by just three independent parameters is illustrated by applying it to the finite problem of a rigid, tilted punch pressed onto a half-plane, and suffering partial slip induced by the application of in-plane tension.

Keywords: Contact mechanics; Elastic material; Friction; Asymptotic analysis; Frictional shakedown

1. Introduction

This paper represents further practical progress in the development of asymptotic procedures for quantifying the state of stress in the crack nucleation regions of incomplete fretting contacts. We have recently (Dini and Hills, 2003) examined the possibility of quantifying the state of stress and interfacial slip displacement in a

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narrow slip band at the edge of an incomplete contact by two characteristic stress fields; one corresponding to normal contact pressure, and the other to the shear traction distribution prevalent. This approach is valuable because it provides a realistic means of reducing dramatically the number of independent variables needed to quantify the nature of the state of stress in the region where cracks nucleate. Furthermore, as these fields may be scaled by two (dimensional) factors to any incomplete contact geometries, it follows that they provide a convenient and efficient means of carrying the results of a suitable test of nucleation life to a geometrically rather different prototype. The solutions described have proved a powerful means of correlating, for example, the local states of stress near a slipping contact edge in (a) the classical Cattaneo–Mindlin contact (Cattaneo, 1938; Mindlin, 1949), (b) a flat-and-rounded contact, and (c) a tilted punch in incomplete contact, as well as a general, numerically solved geometry. There were, however, two limitations in that analysis; (a) the solution was valid only for a fully reversing shear loading, so that no frictional shakedown occurs, and (b) there was no externally imposed bulk tension load. This paper seeks to remedy those defects, hence extending the scope and versatility of the solution considerably, and rendering it a fully practicable tool to correlate crack nucleation conditions. The former is the easier of the two developments to follow, and so we will describe this first.

2. Frictional shakedown

Frictional shakedown is the self-generation of a protective residual interfacial shear-traction which tends to alleviate the shearing traction variation induced by applied loading, and hence to reduce the tendency to slip. It is a phenomenon which is closely allied to the more widely understood form, viz. classical plastic shakedown: like plastic shakedown, it cannot occur under conditions of a fully reversing shear loading, because any residual interfacial shearing traction which is protective when the external shear force is acting in one direction exacerbates the tendency to slip when the shearing force is reversed. It therefore occurs only when the shear loading is unsymmetrical. Further, in incomplete contacts, where the contact pressure falls continuously to zero at the contact edge, it is not possible locally to preserve a residual shearing traction. It follows that the shakedown limit (the load range where stick is maintained *everywhere* within the contact patch) is always zero. At the same time, the development of residual shearing tractions at all points remote from the contact edges is certainly possible, so that the steady-state slip zone may be rather smaller than the initial slip zone in size. This is certainly true of the Cattaneo–Mindlin contact, and which, as before (Dini and Hills, 2003), we will use as the basis of the asymptotic solution we wish to derive.

Fig. 1(a) shows schematically the edge of a semi-infinite incomplete contact, with a coordinate, x, measured from the contact edge. First, as the direct traction is uncoupled the solution for that will be unchanged from our earlier work, i.e. the