Available online at www.sciencedirect.com
SCIENCE DIRECT®
Journal of Mechanics and Physics of solids
53 (2005) 33–48
JOURNAL OF THE MECHANICS AND PHYSICS OF SOLIDS
www.elsevier.com/locate/jmps

Size-dependent sharp indentation—I: a closed-form expression of the indentation loading curve

Yan Ping Cao, Jian Lu*

LASMIS, CNRS FRE 2719, Universite de Technologie de Troyes, 12 rue Marie Curie, BP 2060, 10010 Troyes, France

Received 16 January 2004; received in revised form 15 June 2004; accepted 19 June 2004

Abstract

In this paper, a closed-form expression of the size-dependent sharp indentation loading curve has been proposed based on dimensional analysis and the finite deformation Taylor-based nonlocal theory (TNT) of plasticity (Int. J. Plasticity 20 (2004) 831). The key issue is to link the results of FEM based on TNT plasticity with those obtained using conventional FEM by taking as the effective strain gradient, η, that presented in the work of Nix and Gao (J. Mech. Phys. Solids 46 (1998) 411), thus avoiding large-scale finite element computations using strain gradient plasticity theories. Two experiments carried out on 316 stainless-steel and pure titanium have been used to verify the effectiveness of the present analytical model; the results demonstrate that the present analytical expression of the size-dependent indentation loading curve corresponds very well to the experimental indentation loading curve. The empirical constant, α, in the Taylor model estimated from the experimental data has the correct order of magnitude. Also, the results presented in this part can be further applied to establish an analytical framework to extract the plastic properties of metallic materials with sharp indentation on a small scale where the size effect caused by geometrically necessary dislocations is significant. This will be discussed in detail in the second part of the paper.

Keywords: Taylor-based nonlocal theory of plasticity; Dimensional analysis; Closed-form expression; Size-dependent indentation loading curve; Plastic properties

*Corresponding author. Tel.: +33-3-2571-5650; fax: +33-3-2571-5675.
E-mail address: lu@utt.fr (J. Lu).

0022-5096/$ - see front matter
doi:10.1016/j.jmps.2004.06.005

1. Introduction

Many experiments have demonstrated that metallic materials have significant size effects when the characteristic length associated with nonuniform plastic deformation is at micron scale. Here we mainly refer to the size effects observed in nano or micro-indentation tests (Fleck et al., 1994; Stelmashenko et al., 1993; De Guzman et al., 1993; Ma and Clarke, 1995; Nix and Gao, 1998; Elmustafa and Stone, 2003; Zhao et al., 2003), based on the consideration that much effort has been devoted recently to the development of systematic methods to extract the mechanical properties of materials with depth-sensing instrumented indentation. Without considering size effects, much work has been presented to extract material properties by means of micro-indentation tests. For single indenter algorithms, Giannakopoulos and Suresh (1999) have proposed a systematic framework to obtain elastic–plastic properties within the context of small-strain finite element analysis. Since then, more comprehensive work has been carried out by Dao et al. (2001) based on dimensional analysis and large deformation FEM. Although in the case of many engineering metals, a single set of plastic properties can be extracted from single indenter algorithms, the results were found to be sensitive to small experimental errors. More recently, this problem has been thoroughly explored by Capehart and Cheng (2003). Their results show that 1% noise levels preclude the accuracy of the plastic properties identified, such as the strain hardening exponent. Based on the fact that the plastic properties extracted from a single P–h curve are sensitive to small experimental errors, two comprehensive studies have been carried out recently (Bucaille et al., 2003; Chollacoop et al., 2003). Dual sharp indenter algorithms were devised to improve the accuracy of the identified results. The authors (Cao and Lu, 2004) have studied the stability of the dual indenter algorithms by introducing the concept of the condition number. In that article, ill-conditioned cases in the inverse problem are examined and corresponding regularization schemes proposed. Since the selection of tip apex angles can change the properties of the sensitivity matrix in the inverse problem, guidelines have been developed for optimal combinations of tip apex angles and verified by means of numerical examples.

It should be emphasized that when using dual sharp indenter algorithms to extract the plastic properties of materials, all that are needed are the Young's modulus and the indentation loading curves which are the basis of reverse algorithms. During the loading procedure, the p–h response of a homogeneous elastic–plastic material to sharp indentation generally obeys the following relationship which was found to be a natural outcome of dimensional analysis (Cheng and Cheng, 1998a, b).

$$p = Ch^2, \tag{1}$$

where C is the loading curvature, constant for given material properties and independent of the indentation depth.

However, Eq. (1) will not be true if the size effects are significant. Here, we will mainly discuss the size effect induced by geometrically necessary dislocations (GND) (Stelmashenko et al., 1993; De Guzman et al., 1993; Ma and Clarke, 1995; Nix and Gao, 1998). Dao et al. (2001) have suggested a large indentation depth to exclude