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Mechanics of Materials 37 (2005) 1180–1197
MECHANICS OF MATERIALS
www.elsevier.com/locate/mechmat

Electromechanical model of periodic cracks in piezoelectric materials

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Received 26 August 2004; received in revised form 6 April 2005

Abstract

The electro-elastic problem for a periodic array of cracks in a piezoelectric medium subjected to coupled electromechanical loads is investigated. The mixed boundary value problem, which is formulated directly in terms of the crack surface displacements and electrical potentials, results in a system of hyper-singular integral equations in which the unknown functions are the crack surface displacement and electric potential. Numerical results include the crack surface displacement and the stress and electric intensity factors for the entire range of possible periodic crack spacing and medium size. The central contribution of this paper is the development of an analytical model that predicts crack-spacing effect. The resulting model is validated by a 2D finite element analysis.

Keywords: Piezoelectric materials; Fracture mechanics; Periodic cracks

1. Introduction

Piezoelectric ceramics are preferred materials for a wide variety of smart structures, and electronic and mechatronic devices due to their pronounced piezoelectric, dielectric, and pyroelectric properties. Although these materials have considerable technological capabilities, they are limited by fatigue and fracture.

Recent work in the area of fracture mechanics has been focused on determining crack tip fields in anisotropic piezoelectric materials and explaining the effect of electric field on the fracture behavior of piezoelectric materials. A lengthy literature has been developed for the fracture mechanics of piezoelectric materials. The Stroh formalism in elastic material has been extended to piezoelectric materials to obtain the solution for an anti-plane crack in piezoelectric materials (Suo et al., 2002).

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doi:10.1016/j.mechmat.2005.05.003

Pak obtained the solution for an anti-plane crack in piezoelectric materials (Pak, 1992). A general solution for thermopiezoelectrics with various holes under thermal loading has been provided (Qin, 2000). Electric breakdown-resistant dielectric and ferroelectric ceramics was discussed in (Suo, 2003). An electrical non-linearity concept in the fracture of piezoelectric materials was proposed cussed by Gao et al. (1997), and by Fulton and Gao (2001). Energy change in an infinite body due to the introduction of an elliptical cavity having an electric field within it was calculated and discussed by McMeeking (2004). Theoretical results and experimental observations in piezoelectric materials have been reviewed by Zhang et al. (2002), and by Zhang and Gao (2004).

Although a variety of challenging issues related to certain crack problems in the piezoelectric materials have been addressed, one of the remaining problems that needs to be fully understood is that of a periodic array of parallel cracks in such media subjected to coupled electromechanical loads. Past experience suggests that cracks in a medium may be either a single dominant crack or a roughly regular array of periodic cracks (Grot and Martyn, 1981; Nied, 1987; Rizk, in press; Schulze and Erdogan, 1998; Timm et al., 2003; Erdogan and Ozturk, 1995). Accordingly, there is a need to investigate the effects of multiple cracks on the mechanical and electric failure phenomena in piezoelectric media. A piezoelectric medium with two surface cracks has been discussed by Li and Lee (2004). Solution for the periodic array of non-collinear cracks in piezoelectric materials was not available at the present time.

The focus of this paper is the development of a mathematical model to predict the length scale for the spacing of transverse cracks that form in a piezoelectric medium subjected to a coupled electro-mechanical external loading condition. The resulting model is validated by a 2D finite element analysis. The paper is organized as follows: Section 2 describes the basic equations and boundary conditions for a two-dimensional piezoelectric layer. Section 3 develops a mathematical model for a row of periodic cracks (embedded cracks or edge cracks) in the strip subjected the coupled external electromechanical loading. In this section, the model is verified by the finite element solution. For the completeness of the research, Appendix A describes the solution technique for the mode II fracture of the piezoelectric materials with a periodic array of cracks.

2. Description of the problem

Investigated are the interactions of a row of cracks periodically located in a piezoelectric material layer shown in Fig. 1(a). The upper tips and the lower tips of the cracks terminate at $y = b$ and $y = d$, respectively. Crack length is $2a$. The principal axes of the each layer are directed along the x and y axes. The thickness of the piezoelectric layer is denoted as h.

With reference to the coordinate system (x, y) set at the top surface and the poling direction of the medium is parallel to the positive x direction, the elastic stresses and electric displacements are related to the strain and electric fields through:

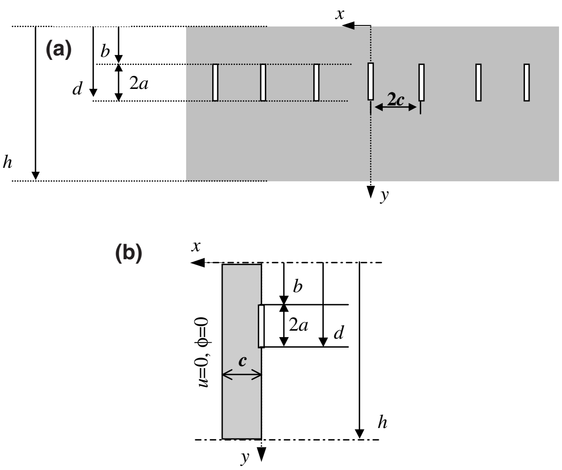

Fig. 1. (a) A piezoelectric material strip with a periodic array of cracks along the x direction. (h is the total thickness of the medium, crack tips are located at $y = b$ and $y = d$. $2c$ is the crack spacing. $d = b+2a$. a is the half crack length. If b is larger than zero the cracks are embedded in the strip. For edge cracks, b equals zero) and (b) a representative unit cell ($0 \leqslant x \leqslant c$, $0 \leqslant y \leqslant h$).