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The compressive response of a stratified fibrous structure

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Abstract

A model for the compressive response of an oriented fiber structure such as paper is derived in this work. The model provides a semi-quantitative description of the stress/strain behavior of a fiber structure. The model accounts for the deformation of fibers during compression, their finite fiber length, their elastic modulus, and their moment of inertia. Using a previously describe micro-indentation technique, the predicted impact of variables used in the model on the compressive response of paper is verified. Results consistent with the model are found including a strong dependence of the tangent modulus on apparent density of the sheet, and a relative insensitivity of the tangent modulus to fiber length. The three stage behavior of the paper compression described by Rodal [Soft-nip calendering of paper and paperboard. Tappi Journal 72 (5), 177–186] was also accounted for in the model.

Keywords: Compressive modulus; Tangent modulus; Compression model; Finite fiber length; Fiber orientation

1. Introduction

Paper is typically composed of fibers that are obtained from wood by a mechanical or chemical pulping process. The chemical pulping operation removes most of the undesirable lignin from the wood leaving fibers composed primarily of cellulose along with other carbohydrates (i.e. hemi-celluloses) and small amount of other materials. This work focuses on paper sheets made with chemically pulped fibers. Paper is formed by dispersing wood pulp fibers in water at a solids content of about 0.5% and draining the water from the suspension through a fine screen to form a wet web. Upon drying, a porous structure of fibers is created. The fibers are connected by hydrogen bonds and arranged in a random fashion within the plane of the sheet. The long axes of the fiber, which commonly have an aspect ratio of ~60, are primarily oriented parallel to the plane of the sheet with

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minimal interlayer crossing. This results in what may be called a stratified fibrous structure. The mechanical properties of the paper sheet are dependent primarily on the degree of inter-fiber bonding, the organization of the fiber structure, and the fiber's morphological and polymer characteristics. The resulting sheet is also anisotropic in three dimensions resulting from the anisotropic nature of the fibers, preferential alignment of the fibers during sheet forming, and the stress imparted to the paper during drying. The three dimensions in the paper sheet are described by specific terms. The two in-plane dimensions are described by the machine direction (MD) and the cross machine direction (CD). The *z*-direction is normal to the planar surface and the mechanical properties along this axis are of interest in this work. In particular, this work develops a model for the compressive response of paper in the *z*-direction. The model takes into account the general properties of the fiber structure in a paper sheet. It explicitly accounts for the bending of the fiber during compression and the finite fiber length of typical papermaking fibers.

Investigations into the *z*-direction compressive properties of paper have been primarily motivated by an interest in improving the printability or calendering performance of paper. While the compressive characteristics play an important role in these two processes, they also play a role in determining friction characteristics, roll quality, sheet softness (tissue), and performance in converting and intermediate processing operations. The importance of the *z*-direction properties has led to the development of various experimental techniques to measure the compressive characteristics (Bristow and Kolseth, 1986; Baysung, 1964; Gavelin, 1949; Ivarsson, 1956; Jackson and Ekstrom, 1964; Pawlak and Keller, 2003; Pfeiffer, 1996; Ratto and Rigdahl, 1998; Roehr, 1955; Stenberg et al., 2001; Ting et al., 2000; Yamauchi, 1989), as well as a number of models that can be used to characterize the compressive properties of paper (Heikkila, 1997; Pfeiffer, 1981; Schaffrath and Gottsching, 1991).

Many of the previously developed models have used combinations of springs and dashpots to describe the paper compressive characteristics. While these models are useful in characterizing compression data in terms of simple parameters, they do not provide insight into how different papermaking variables and fiber properties may affect the compressive characteristics. In contrast, the model described in this work takes into account the random nature of the fiber network, the fibers' finite fiber length, their elastic properties, and coarseness. This allows for one to computationally evaluate the effect of changing these variables on the resulting paper compressibility.

2. A model for the compressive behavior of a fibrous structure

The mechanical properties of a fibrous structure are strongly influenced by the fiber-to-fiber contacts. van Wyk (1964) described the compressive behavior of a fiber structure with fibers oriented randomly in three dimensions. This investigator used fiber bending to describe the stress/strain relationship. In modeling the compressive response, van Wyk determined a number of parameters for the fiber network. Among these parameters were the mean distance between contacts, the mean number of contacts within a given volume, and the mean number contacts per fiber. Komori and Makishima (1977) later showed a generalized analysis of these parameters for a fiber network with arbitrary fiber orientation. These researchers showed that the generalized analysis agrees with the specific cases used by van Wyk (1964) (i.e. fiber randomly oriented in the three dimensions) and by Kallmes and Corte (1960) (i.e. a two-dimensional sheet of randomly oriented fibers). Kallmes and Corte verified their analytical findings experimentally, which is much simpler in the case of a two-dimensional sheet. Neckar (1997) presented an extension of van Wyk's model that included the volume occupied by the fibers during compression.

In the following analysis, the compression of a three-dimensional paper structure will be examined. The analysis is based on the approach used by van Wyk, using his general equations to fit constraints that are more appropriate with paper structures. The analysis by Komori and Makishima, for the specific case where the fibers are