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Longitudinal deformation of fibre reinforced metals: influence of fibre distribution on stiffness and flow stress

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Abstract

A computational analysis of the longitudinal deformation of continuous fibre reinforced metals is presented. Elastic and elastic–plastic matrix behaviour are considered. Analytical approaches are confronted with finite element analyses (FEA) for varying fibre distributions, ranging from single fibre unit cells to complex cells. Analysis of microfields shows that the main cause for deviation from the equistrain rule of mixtures is a stiffening effect of matrix confinement when surrounded by touching fibres arranged as "rings". Comparison with FEA shows that Hill's [J. Mech. Phys. Solids 12 (1964) 199, 213] bounds, although best possible in terms of volume fraction, are of limited value in so far as Hill's upper bound lies far above any practical limit for a fibre reinforced material, whereas Hill's lower bound loses its bounding property when extended to non-linear behaviour via an incremental scheme. This latter effect can be corrected by changing slightly Hill's derivation in a way that preserves the bounding property. Finally, implications are given for the derivation of in situ matrix flow stress curves from experimental tensile curves on fibre reinforced composites. It is suggested that linear three-point bounds can in practice be used for this purpose.

Keywords: Fibre reinforced composites; Inelastic behaviour; Bounds; Stiffness; Finite element modelling; In situ matrix flow stress

1. Introduction

Assessing the flow stress or stiffness of a unidirectional fibrous material, in which all phases are cylindrical (Hashin, 1983), parallel to the fibres (i.e., in axial loading) is a trivial problem as long as "engineering precision" is sufficient: the equistrain rule-of-mixtures (RoM) applied to the flow stress or stiffness provides adequate precision. The underlying reason is that, when the composite is stressed parallel to aligned fibres, stress and strain are (i) relatively uniform and (ii) far higher along the fibres than in other directions. Hence the average axial stress in each phase roughly equals that which is measured in a tensile bar of the same material taken to the same axial strain ϵ as the composite, i.e.,

$$\sigma_c = V_1\sigma_1 + V_2\sigma_2 \tag{1}$$

with the corollary that, for elastic deformation:

$$E_c = V_1E_1 + V_2E_2 \tag{2}$$

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where E denotes Young's modulus, V the volume fraction, and subscripts c, 1 and 2 stand for the composite, Phase 1 and Phase 2, respectively. Eq. (1) and its corollary Eq. (2), adapted when necessary to account for the presence of residual stress in each phase (due for example to thermal contraction mismatch between the phases, see, e.g., de Silva and Chadwick, 1969; Garmong, 1974; Tyson, 1975), are widely known to provide good descriptors of longitudinal composite deformation.

Occasionally, there may arise the need to obtain better precision in linking the longitudinal flow stress of composites with that of its phase constituents. One instance is found with the inverse problem, namely extracting individual phase flow properties from the measured composite stress–strain curve. This has for example been done with fibre reinforced metals to expose size effects in metal plasticity (Kelly and Lilholt, 1969; Isaacs and Mortensen, 1992; Bystricky et al., 1999). This inverse problem is of interest because, with fibre reinforced metals stressed along their axis, phase stresses are relatively uniform in both elastic and elastoplastic deformation (Hill, 1964a,b; Mulhern et al., 1967; Dvorak, 1991; Brockenbrough and Suresh, 1990; Brockenbrough et al., 1991; Böhm et al., 1993; Böhm and Rammerstorfer, 1994). In essentially all other configurations (transversely stressed or laminated fibrous composites, short-fibre or particulate composites...) the matrix stress and strain distributions are highly non-uniform and triaxial, such that the measured average stress has less fundamental meaning without a fully accurate mechanical model (which itself requires knowledge of the in situ phase properties as its input). The reason why higher precision is then required is that, when back-calculating the matrix flow stress from that of a long-fibre composite, the load borne by the generally very stiff fibres far exceeds that which is carried by the matrix. The back-calculated matrix flow stress then results from the subtraction of two far larger numbers (Eq. (1)). Even very minor error in Eq. (1) then causes major uncertainty in the back-calculated matrix flow curve.

Mechanics-related deviations in the composite flow stress or modulus from the rule of mixtures arise from the presence of lateral stresses, themselves due to incompatibility in lateral deformation between the matrix and the reinforcement. In elastic deformation, this is the case whenever the Poisson ratio differs between matrix and fibres: the two phases then exert a mutual constraint on each other that raises the composite stiffness above the value predicted by the RoM, such that:

$$E_{\Delta\nu} \equiv E_c - (V_1E_1 + V_2E_2) \geqslant 0 \tag{3}$$

Hill has derived bounds for the longitudinal stiffness of unidirectional fibrous materials, and hence for $E_{\Delta\nu}$ (Hill, 1964a):

$$\frac{4V_1V_2(\nu_1-\nu_2)^2}{V_1/k_2+V_2/k_1+1/G_1} \leqslant E_{\Delta\nu} \leqslant \frac{4V_1V_2(\nu_1-\nu_2)^2}{V_1/k_2+V_2/k_1+1/G_2} \tag{4}$$

where ν designates the Poisson ratio, $G = E/(2(1+\nu))$ designates the shear modulus, $k = E/(2(1+\nu)(1-2\nu))$ the plane strain bulk modulus, and the indices 1 and 2 designate the soft and hard phase, respectively. The lower bound corresponds to the longitudinal modulus of an elementary cylindrical composite consisting of a single fibre of the stiffer phase with circular section embedded in a circular cylindrical shell of the more compliant phase. This simple arrangement yields the same result as the composite cylinder assemblage (CCA) proposed by Hashin and Rosen (1964). The upper bound is constructed by inverting the phase properties.

The Hill bounds, although tight in absolute numbers, are relatively slack with regard to the possible error in "back-calculation" of the flow stress of a soft matrix. In particular, it is intuitively clear that the solution for the upper bound is largely above that of any typical (stiff elastic) fibre reinforced composite, since it describes a hard interconnecting matrix with compliant fibres.

More elaborate models can only be constructed by incorporating information on the spatial arrangement of the two phases. Higher order bounds, e.g., Milton (1982), Torquato (1991), Torquato and Lado (1992) and estimates (Torquato, 1998) are constructed by incorporating statistical information on the arrangement of the phases, e.g., on random arrangements of hard fibres in a soft matrix. Three-point bounds are much