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A story about estimation of a random field of boulders from incomplete seismic measurements

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Abstract

This paper reports on the statistical interpretation of seismic diffraction measurements of boulder locations. The measurements are made in a corridor along the planned tunnel line for the later realized bored tunnel through the till deposits under the East Channel of the Great Belt in Denmark. The investigation was made in 1987–1988 on the initiative of A/S Storebæltsforbindelsen in order to prepare its tendering for the bored tunnel. The purpose was to make a prediction of the density of boulders of maximal dimension above any specified length. The properties of the seismic measuring device and its graphical registrations on seismograms do not make a proper interpretation possible without detailed knowledge about the joint distribution of the primary dimensions of the boulders. Therefore, separate measurements were made of the dimensions of boulders deposited visibly on the cliff beaches of the Great Belt. The surprisingly simple results from the analysis of the data from this separate measuring program are reported in a companion paper. Based on the revealed geographical universality of the joint dimension distribution it is anticipated that the same distribution is applicable for the till deposits along the tunnel line. By use of this important distribution information and of the observed homogeneity of the seismic point source field together with the physical properties of diffraction it became possible to make the wanted prediction. During the excavation, the found boulders were counted and measured. These direct observations on site confirmed that the prediction was quite good.

Keywords: Boulders; Diffraction; Field investigations; Glacier till; Predictions; Probabilistic models; Tunnel boring; Seismic surveys; Statistical inference

1. Introduction

Like the story told in [1] this is an interesting story that has only been told in reports of limited circulation [2,3] and in summary form in [4]. In order to investigate the conditions for making a bored tunnel below the East Channel, A/S Storebæltsforbindelsen (SBF) in 1987 took the initiative to let the Danish Geotechnical Institute (DGI) make a seismic scanning for boulders in the till in a 400 m wide and 2500 m long corridor along the planned tunnel line at the western side of the East Channel. The geology within this corridor is roughly a 16 m thick till formation containing boulders which has been deposited by glacial activity on top of a marine deposit of marl that contains no boulders. At the time of the investigation the two parallel tunnel tubes were supposed to be bored with a slope that takes the tubes through the till down to the marl over a distance of about 1000 m. Extensive descriptions of the final project are published in [5]. However, the position of the tunnel is not relevant for the investigations reported in this paper, because in a deterministic sense it was not possible to observe all boulders and to determine the size of the boulders that actually would be met by the boring machines. Instead, the investigation aimed at making a statistical prediction of how densely and with which sizes boulders could be expected to be met during the boring. Therefore, the search for boulders was extended to the entire till volume within the selected corridor making the hypothesis to be tested that this till volume is representative for the statistical occurrence and size of the boulders within the much smaller tube shaped till volumes planned to be removed be the boring machines.

A sound sending and receiving device was developed for the purpose of this investigation at DGI by Tirey [6]. To use the device it is placed in the water surface behind a boat that moves with constant speed. The sound signals from

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the device travel through the water and the till deposit and get diffracted if they meet a boulder of vertical size larger than about a quarter of the wave length of the sound. Reflections from the soil layer surfaces as well as diffraction signals from boulders are observed by the device and are plotted on a seismogram. A hyperbolically curved mark on the seismogram indicates a diffraction signal from a boulder as if it comes from a point source at the center of the boulder. The sole information obtained is the distance from the device to the center of the boulder.

Points of boulder positions were registered along several approximately parallel lines traveled by the boat with the seismic device in operation. However, these registrations are incomplete both with respect to the position orthogonal to the sail line and due to a lack of boulder size information. As mentioned, the size information is solely that the vertical dimension of the observed boulder is larger than a certain limit of resolution δ about one quarter of the wave length of the transmitted sound ($\delta \approx 0.5$ m). In the following such boulders are called 'qualified'. Moreover, not all boulders in the scanned volume seem to be detected. This incompleteness raises an interesting problem of data analysis.

The first part of the analysis consists of applications of statistical methods to test the hypothesis that the registered points along the sail line are placed as points of a homogeneous Poisson point field. The analysis shows that this hypothesis cannot be rejected. However, comparison of the intensity estimates for the different sail lines reveals fluctuations beyond what can be explained by the statistical uncertainty of the estimation. Therefore, these fluctuations must in part be attributed to randomly missing observations of qualified boulders. Even though the boulder locations were read and digitalized directly by an experienced observer at DGI, the observer's ability to see the marks on the seismograms fluctuates randomly. This is partly due to superposed noise on the seismograms related to varying sea conditions and partly due to the different states of fatigue of the observer. By adopting a plausible probabilistic thinning

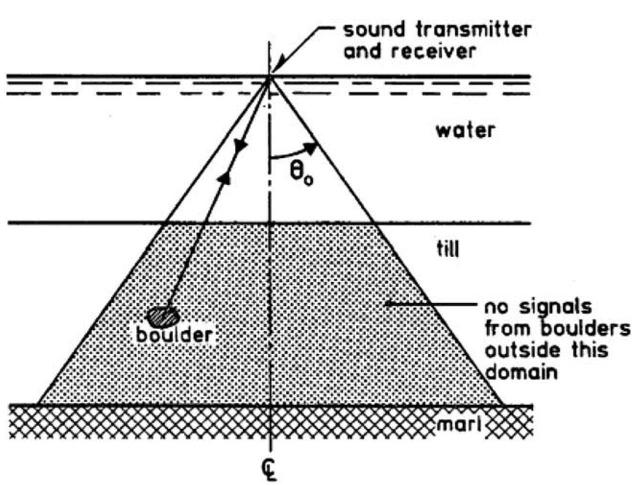

Fig. 1. Illustration of idealized model of the directional sensitivity of the seismic device.

model and applying maximum likelihood estimation, the resulting underestimation bias of the intensity can be evaluated.

The conclusion of the statistical analysis is that the diffraction signals appear as if they originate from a homogeneous point field in the vertical plane of the sail line except for those signals that come from a narrow neighborhood of the boundary between the till deposit and the marl deposit underneath the till. At this boundary, the intensity jumps up by a factor of about three. This anomaly turns out not to be caused by a larger boulder density at the marl surface. The jump in intensity is explained by the physical theory of sound wave propagation and diffraction if it is assumed that the sound transmitting and receiving device is directionally sensitive. With sufficient accuracy for the considered problem it is assumed that sound rays are received by the device only if they propagate within a circular cone with vertex at the device and top angle $2\theta_0$, see Fig. 1. In other words, when the device is at rest in the sea surface no registrations originate from boulders placed outside this sensitivity cone. Moreover, it is assumed that within the cone there is a direction-independent threshold of sound pressure, below which no signal can be detected by the device. From physics, it is known that the diffracted sound pressure is directly proportional to the volume of the diffracting boulder and inversely proportional to the distance between the device and the center of the boulder. This is sufficient to determine the angle θ_0 by simply requiring that the point field intensity jump at the marl surface becomes removed. Hereby, the value $\theta_0 = 35°$ is estimated with a coefficient of variation less than 2%. Since the boat moves forward, the scanned volume along the sail line gets a trapezoidal cross-section defined by the sensitivity cone.

To obtain the density γ of qualified boulders per volume unit of till from the intensity κ of diffracted signals per area unit in the vertical plane along the sail line, it is necessary to use information about the joint distribution of the three main dimensions of the boulders. This information is obtained from statistical analysis of extensive measurements of visible boulders on the cliff shores of the Great Belt at widely separated localities. This study, described by Ditlevsen [1], reveals that within the boulder deposits of the Great Belt region a surprisingly simple joint probability distribution of the three main dimensions applies. Using this probability distribution together with the geological experience supported assumption that the vertical dimension is the smallest of the three dimensions leads to an assessment of the ratio γ/κ as a function of δ.

A part of this assessment is based on a requirement of consistency between the physical principles and the boulder volume distribution. Since the small boulders are more round ('ellipsoid'-shaped) while the larger boulders are more box-shaped, there is a shape factor on the product of the three principal dimensions to obtain the volume. The factor is between about $\pi/6 \approx 0.52$ and up to about 1