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Unsteady flows of a binary mixture of incompressible Newtonian fluids in an annulus

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Abstract

This paper is concerned with applying the mixture theory of two chemically inert incompressible Newtonian fluids to some simple unsteady flows in the annular region between two infinitely long coaxial cylinders. The equations governing the motion of the binary mixture under discussion are reduced to a system of coupled partial differential equations. With the help of finite Hankel transforms, the exact solutions of these equations are obtained in series form for the following three problems: (i) unsteady axial Couette flow in an annulus, (ii) unsteady Poiseuille flow in an annulus, (iii) unsteady circular Couette flow in an annulus.

Keywords: Mixture; Newtonian fluid; Unsteady flow; Hankel transform

1. Introduction

Because of their importance in various industrial applications, flow of a mixture of two fluids has been the subject of numerous theoretical studies. A familiar example is an emulsion, which is the dispersion of one fluid within another fluid. Typical emulsions are oil dispersed within water or water within oil. Emulsions of this type are of practical interest because synthetic fluids are more toxic than mineral oils and are uneconomical to use in applications requiring large quantities of lubricant, for which examples are metal working, mining, cutting and hydraulic fluids. Another example where the mixture of fluids plays an important role is in multigrade oils. To enhance the lubrication properties of mineral oils, such as viscosity index, polymeric-type fluids are added to the base oil [1].

The first general statement of the balance and conservation equations for a continuum theory of mixtures, origins of which are based on the work of Fick [2], was presented by Truesdell [3]. After his pioneering work, the theory of mixtures have drawn considerable attention, and a good amount of literature has grown up around this subject. For the historical development and details of the mixture theory as well as extensive references to previous work in this field, the reader should consult Bowen [4], Atkin and Craine [5,6], Bedford

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and Drumheller [7], several appendices in the recent edition of Rational Thermodynamics by Truesdell [8], and Rajagopal and Tao [9].

Much of the developments in the theory of mixtures have been mainly restricted to the general formulations of the basic equations and constitutive models . Very little efforts have so far been made to apply the theory to practical problems. One main reason for this lacuna is that there are serious difficulties with regard to specifying boundary conditions in mixture theory [9]. For instance, the assumption that the total mixture velocity be zero on the boundary is not equivalent to requiring the constituent velocities are zero; the latter implies the former but not vice-versa. If one assumes the former, and it is only this that is known in a homogenized sense, then one will not have enough boundary conditions. It is possible to get out of this difficulty in certain special problems by appealing to physical arguments. In the case of a solid–fluid mixture for the situation in which a mixture boundary is in a saturated state, Rajagopal et al. [10] derived a thermodynamic condition which they substituted in place of the missing boundary condition. Gandhi et al. [11] studied two boundary value problems corresponding to the diffusion of a fluid through an elastic solid using the condition derived on the basis of thermodynamics. The same problems as that investigated in [11] solved by Tao and Rajagopal [12] using a boundary condition that stems from a purely mechanical consideration.

Adkins [13] formulated constitutive equations for the stresses in each constituent, and for diffusive force. He also examined some steady-state flows of compressible mixtures of non-Newtonian fluids. The mixture of two compressible Newtonian fluids together with full thermodynamical restrictions was first considered by Green and Naghdi [14]. Their results were used by Mills [15] studying a binary mixture of incompressible Newtonian fluids and applied to problem of helical flow. Craine [16] examined the flow induced by the steady oscillations of an infinite plate in a mixture of two incompressible Newtonian fluids. In his subsequent study, he considered the same problem for a binary mixture of incompressible Newtonian hemihedral fluids [17]. Beevers and Craine [18] extended the list of known solutions for a mixture of two incompressible Newtonian fluids and discussed in more detail methods for evaluating the response functions. Later some exact solutions for the flow of a binary mixture of incompressible Newtonian fluids were presented by Göğüş [19–24]. Several problems relating to the mechanics of oil and water emulsions have been considered within the context of the mixture theory by Al-Sharif et al. [25], Chamniprasart et al. [26], and Wang et al. [27]. In a recent paper, Barış and Dokuz [28] obtained the exact solutions in series form for the flow of a binary mixture of incompressible Newtonian fluids in a rectangular channel.

In the present paper, we have sought special semi-inverse solutions of the equations of motion governing the unsteady flows of a binary mixture of incompressible Newtonian fluids in the annular region between two infinitely long coaxial cylinders. We have obtained the exact solutions in series form for the velocity fields by means of finite Hankel transforms. These solutions are important not only as solutions of fundamental fluid-dynamic flows, but also serve as accuracy checks for the approximate solutions such as numerical, asymptotical and empirical.

2. Basic theory

We present the following short exposition of the basic balance laws and the appropriate constitutive theory for a binary mixture of chemically inert incompressible Newtonian fluids for the convenience of the readers. More detailed information can be found in the review articles by Atkin and Craine [5,6] and Bedford and Drumheller [7].

We consider a mixture of two interacting constituents, each of which is regarded as a continuum; we assume that each constituent $s^{(\beta)}$ is a chemically inert incompressible Newtonian fluid. Throughout this paper β takes the values 1 and 2. At any time t each point x in the mixture is occupied simultaneously by one particle from each $s^{(\beta)}$. This is a basic assumption of the continuum theory of mixtures and its validity requires that the mixture appears homogeneous when viewed on the scale of the applied disturbance. If $\mathbf{v}^{(\beta)}$ denotes the velocity vector of the βth constituent, the material time derivative $\mathrm{D}^{(\beta)}/\mathrm{D}t$ is defined by

$$\frac{\mathrm{D}^{(\beta)}}{\mathrm{D}t} = \frac{\partial}{\partial t} + \mathbf{v}^{(\beta)} \cdot \nabla, \tag{2.1}$$

where ∇ is the gradient operator.