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Electroelastic analysis for a Griffith crack interacting with a coated inclusion in piezoelectric solid

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Abstract

A Mode III Griffith crack interacting with a coated inclusion in piezoelectric media is investigated. The crack, the coated inclusion are embedded in an infinitely extended piezoelectric matrix media, with the crack being along the radial direction of the inclusion. In the study, three different piezoelectric material phases are involved: the inclusion, the coating layer, and the matrix. A far-field loading condition is considered. During the solution procedure, the crack is simulated as a continuous distribution of screw dislocations. By using the solution of a screw dislocation near a coated inclusion in piezoelectric media as the Green function, the problem is formulated into a set of singular integral equations, which are solved by numerical method. The stress and electric displacement intensity factors are derived in terms of the asymptotic values of the dislocation density functions evaluated from the integral equations. Numerical examples are given for various material constants combinations and geometric parameters.

Keywords: Dislocation; Coated inclusion; Crack; Stress and electric displacement intensity factor; Piezoelectric

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Nomenclature

a radius of an inclusion or inside radius of a coating layer of an inclusion
A_1, A_2 constants which is obtained with the solution of a screw dislocation interacting with a coated inclusion in piezoelectric media
$A_{-1}^{(2)}$, $A_1^{(2)}$ constants for stress of the intermediate layer which is obtained with the solution of a three-phase piezoelectric cylinder model under a field–field anti-plane mechanical load and a far-field in-plane electrical load
b outside radius of a coating layer of an inclusion
B_1, B_2 material constants which is obtained with the solution of a screw dislocation interacting with a coated inclusion in piezoelectric media
$B_{-1}^{(2)}$, $B_1^{(2)}$ constants for stress of the intermediate layer which is obtained with the solution of a three-phase piezoelectric cylinder model under a field–field anti-plane mechanical load and a far-field in-plane electrical load
$A^{(1)}$, $B^{(1)}$ constants for stress of the inclusion which is obtained with the solution of a three-phase piezoelectric cylinder model under a field–field anti-plane mechanical load and a far-field in-plane electrical load
$A_{-1}^{(3)}$, $B_{-1}^{(3)}$ constants for stress of the matrix which is obtained with the solution of a three-phase piezoelectric cylinder model under a field–field anti-plane mechanical load and a far-field in-plane electrical load
b_{-uk}^{III}, $b_{-\varphi k}^{\mathrm{III}}$ coefficient functions associated with the dislocation location and Burgers vector
b_z Burgers vector for the displacement discontinuity at the dislocation core
b_φ Burgers vector for the electric potential discontinuity across the slip plane at the dislocation core
$b_z(\xi)$, $b_\varphi(\xi)$ charged dislocation density functions along the crack line
$c_{44}^{(i)}$ elastic constants
D_∞ far-field in-plane electric loading
D_y electric displacement
$e_{15}^{(i)}$ piezoelectric coefficients
E_x, E_y in-plane electrical field
$F(t)$, $G(t)$ bounded function
$G_{z\varphi}(t,s)$ function of the regular part of the kernel
$\kappa_{11}^{(i)}$ dielectric constants
K_σ stress intensity factor
K_D electric displacement intensity factor
$p_{\sigma(x)}$ mechanical traction function along the plane $\theta = 0$
$p_D(x)$ electric traction function along the plane $\theta = 0$
s_i the set of discrete integration points
t_1 distance of the crack left tip to the center of the inclusion
t_2 distance of the crack right tip to the center of the inclusion
t_i collocation points