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Fuzzy systems and controllers: Lyapunov tools for a regionwise approach

P. Borne[a,*], J.-Y. Dieulot[b]

[a] *LAGIS- Laboratoire d'Automatique Génie Informatique et Signal, UMR CNRS 8136, Ecole Centrale de Lille, Cité Scientifique, 59651 Villeneuve d'Ascq Cedex, France*

[b] *LAGIS- Laboratoire d'Automatique Génie Informatique et Signal, UMR CNRS 8136, EPU/IAAL, Cité Scientifique, 59655 Villeneuve d'Ascq Cedex, France*

Abstract

This paper reviews a class of tools which can help to check the stability of fuzzy linguistic controllers which is a difficult task due to their regionwise structure. The application of conventional methods when considering a fuzzy controller as a global nonlinear controller often leads to conservative stability results. The use of appropriate continuous or piecewise continuous Lyapunov functions allows to address the stability of nonlinear systems under fuzzy control while taking into account the local structure of the fuzzy controllers. Some methods are presented which enforce the global Lyapunov function to decrease when leaving one subspace for another closer to the equilibrium, using concentric domains and providing less conservative conditions.

Keywords: Lyapunov functions; Stability; Fuzzy control; Piecewise affine systems

1. Introduction

Fuzzy linguistic controllers (FLC) aim at representing the control knowledge of an experienced operator with the help of fuzzy rules, and has proved to be efficient in practice [26,29]. Despite the increasing number of applications, the development of systematic methods to analyze and to design fuzzy control systems remains an open research field. A main

* Corresponding author.
E-mail address: pierre.borne@ec-lille.fr (P. Borne).

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problem when handling the stability analysis of fuzzy linguistic controllers is that the control is generally local, i.e. it is possible to partition the premise space (generally the state space) into regions for which the controller parameters and/or structure are region-dependent [10,22,43]. The fuzzy controllers can thus be continuous or discontinuous, depending on the operators which are used and on the rule base.

Fuzzy controllers for which the consequent part is a linear state model, known as Takagi–Sugeno (TS) fuzzy systems [37], have received much attention in the last years. These models allow to represent a large class of nonlinear systems (e.g. [44] and related papers), and can be represented by polytopic linear systems. Sufficient stability conditions for this class of systems rely on the existence of a common quadratic Lyapunov function for all subsystems, which allows then to design state feedback [38,39]. The search for controllers gain and the Lyapunov function can be stated as a convex optimization problem in terms of linear matrix inequalities (LMI) for which efficient solving methods and software exist [4].

Unfortunately, for linguistic controllers with a regionwise structure or TS subsystems with a high number of subsystems, results turn to be conservative as the state dependence of the membership functions—the coefficients in the polytopic formula—is disregarded. Moreover, many stable systems do not admit a quadratic Lyapunov function; when the number of subsystems increases, the computational cost turns to be important, and the existence or finding of a common Lyapunov function is not guaranteed, albeit the system is actually stable.

To overcome these drawbacks and to take into account the locality of controls, piecewise Lyapunov functions (quadratic or not) have been designed for fuzzy (controlled) systems, linguistic or TS [20,21,31–34]. These tools are flexible and can be used regionwise while keeping the power of Lyapunov functions. The main issue is the behavior of the system at the boundary between two domains. Basically, the Lyapunov functions help to analyze the stability of the system within each domain, while the global energy function should decrease when switching from one domain to another. This has been achieved in [21], where piecewise quadratic Lyapunov functions were parameterized to be continuous at the boundaries of the different cells. The differentiability property of the Lyapunov function for each state can be relaxed using discontinuous Lyapunov functions, meaning that stability can be ensured if these regions were joined properly into the state space [31–33].

The case where regions are embedded into each other is an interesting one, for which the trajectory should be enforced to converge, leaving a domain to enter another closer to the origin. This can be achieved by the tools previously mentioned or by a mechanism involving a set of embedded domains and attractors; in the paper, the latter are computed using the comparison systems theory. The concept of the existence of concentric “independent” regions is in agreement with the philosophy of fuzzy control for which the notions of “far” (from the origin) or “near” are implicitly built into the controller [8,11,13].

The outline of the paper is as follows: after recalling briefly different classes of fuzzy systems and controllers, the regionwise structure of a number of controllers will be emphasized. Some stability tools will be presented, from the search for a common Lyapunov function to piecewise continuous and discontinuous functions, and their use in fuzzy controlled systems analysis. The case of fuzzy controllers with embedded domains will be tackled in the last part, and open questions will be raised.