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Is analytical dynamics a theoretical or an experimental science?

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Abstract

When a mechanical system is subjected to equality constraints, use of the chain rule of differentiation and of generalized inverses of matrices enables us to write the most general possible equation of motion, no use being made of any physical principles, Eq. (8). Then employment of standard physical principles enables us to further interpret the terms in this general equation of motion.

Keywords: Analytical dynamics; Principle of virtual work; Nonholonomic constraints; Nonideal constraints; Pseudoinverse of a matrix

1. Introduction

At first glance it might be assumed that the basic equation of motion of analytical dynamics would be replete with physical assumptions. Yet, this need not be so, as will unfold in the following pages.

Let us hurry to Eq. (8).

2. Statement of the problem

We consider a mechanical system consisting of p-point masses. The mass of the ith particle is denoted m_i, $i=1, 2, \ldots, p$. The position of this particle is the three-dimensional

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column vector x_i, in an inertial Cartesian frame of reference. The system position vector is the column vector x,

$$x = (x_1^{\mathrm{T}}\ x_2^{\mathrm{T}}\ \ldots\ x_p^{\mathrm{T}})^{\mathrm{T}},$$

which is of dimension $3p = n$. We also introduce the mass matrix M, which is of dimension $3p \times 3p$, is a diagonal matrix, is positive definite, and has the masses $m_1, m_2, \ldots, m_p$ down the main diagonal in groups of three, with zeros elsewhere. Assuming that the system is subjected to m equality constraints, involving t, the time, x, and x', the system velocity vector, we wish first to determine *all* the possible equations of motion that are consistent with the constraints [9,10]. Following this, we shall introduce some physical assumptions and specialize the general explicit equation of motion to the physically relevant one.

3. Derivation of the basis equations

Suppose that the system is subjected to m nonholonomic or holonomic equality constraints of the form

$$f_i(x, x', t) = 0, \quad i = 1, 2, \ldots, m. \tag{0}$$

Then by use of the chain rule of differentiation we arrive at a set of m equations that are *linear* in x'', of the form,

$$Ax'' = b, \tag{1}$$

where A is an $m \times n = 3p$ matrix function of x, x', and t, and b is an $m \times 1$ column vector that may depend upon x, x', and t. Experience has shown that the matrix $AM^{-1/2}$ is of great significance [3,6,7], so we shall rewrite Eq. (1) as

$$AM^{-1/2}(M^{1/2}x'') = b. \tag{2}$$

The general solution of this equation may be written in the form [3,7]

$$M^{1/2}x'' = (AM^{-1/2})^{+}b + [I - (AM^{-1/2})^{+}(AM^{-1/2})]z, \tag{3}$$

where z is an arbitrary column vector of dimension $n = 3p$. The matrix $(AM^{-1/2})^{+}$ is the usual pseudoinverse of the matrix $AM^{-1/2}$ [1]. The first term on the right-hand side in Eq. (3), is a particular solution of Eq. (2), and the second is the general solution of the homogeneous equation $(AM^{-1/2})(M^{1/2}x'') = 0$.

We now choose to write the arbitrary vector z in the form

$$z = M^{1/2}a + M^{-1/2}c, \tag{4}$$

where a is a special vector that will be specified later, and c is an arbitrary vector. Eq. (3) then takes the form

$$M^{1/2}x'' = (AM^{-1/2})^{+}b + [I - (AM^{-1/2})^{+}AM^{-1/2}][M^{1/2}a + M^{-1/2}c], \tag{5}$$