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Hemivariational inequalities in thermoviscoelasticity ☆

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Abstract

In this paper we present the existence and uniqueness of the weak solution for a dynamic thermoviscoelastic problem which describes frictional contact between a body and a foundation. We employ the Kelvin–Voigt viscoelastic law which includes the thermal effects and consider the general nonmonotone and multivalued subdifferential boundary conditions. The model consists of the system of the hemivariational inequality of hyperbolic type for the displacement and the parabolic hemivariational inequality for the temperature. The existence of solutions is obtained from a surjectivity result for operators of pseudomonotone type. The uniqueness holds for a large class of operators of subdifferential type satisfying a relaxed monotonicity condition.

Keywords: Contact problem; Hemivariational inequality; Subdifferential; Damping; Nonconvex; Friction

1. Introduction

In this paper we consider the dynamic contact between a linear thermoviscoelastic body and a foundation. The body is assumed to satisfy the Kelvin–Voigt constitutive law with added thermal effects. Our main interest lies in general nonmonotone and multivalued subdifferential boundary conditions. More precisely, it is supposed that on the boundary of

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the body under consideration, the subdifferential relations hold, the first one between the normal component of the velocity and the normal component of the stress, the second one between the tangential components of these quantities and the third one between temperature and the heat flux vector. These three subdifferential boundary conditions are the natural generalizations of the normal damped response condition, the associated friction law and the well-known Fourier law of heat conduction, respectively.

Recently dynamic viscoelastic frictional contact problems with or without thermal effects have been investigated in a large number of papers, see e.g. [1–5,9,12–14,17,18,23,24] and the references therein.

In this paper we investigate a fully dynamic contact problem which consists of the energy-elasticity equations of hyperbolic type together with a nonlinear parabolic equation for the temperature. Because of the multivalued multidimensional boundary conditions, the problem is formulated as a system of two coupled evolution hemivariational inequalities. The latter is embedded into a more general class of problems for second order evolution inclusions. All subdifferentials are understood in this paper in the sense of Clarke and are considered for locally Lipschitz, and in general nonconvex and nonsmooth superpotentials. This allows to incorporate in our model several types of boundary conditions considered earlier e.g. in [4,15,16,19–22,25].

The goal of the paper is to deliver the results on existence and uniqueness of a global weak solution to the system. The existence of solutions is obtained by applying a surjectivity result for operators which are pseudomonotone with respect to the domain of a linear densely defined maximal monotone operator. The existence is accomplished in two steps: first we establish the result for an evolution inclusion with regular initial conditions. In the second step we remove the restrictions on the data and show existence in a general case. In the proof of the existence theorem we combine the approach used in [17] for viscoelastic problems with the one exploited in [18] for hemivariational inequalities of parabolic type. The uniqueness is obtained for a large class of operators of subdifferential type satisfying a relaxed monotonicity condition. For complete proofs of the result presented in this note we refer to [7]. In contrast to [1,3,9,10,14], our approach allows the elasticity operator to be only positive (generally noncoercive). To the best of the authors' knowledge the existence of solutions to the system of hemivariational inequalities in dynamic thermoviscoelasticity has remained an open problem till now. We remark that for linear thermoelastic materials a system of hemivariational inequalities was formulated by Panagiotopoulos in Chapter 7.3 of [22]. However, the regularity hypotheses on the multivalued terms were quite unnatural and the data were assumed to be very regular (cf. [22, Proposition 7.3.2]).

We now recall some notation needed in the sequel. We denote by $\mathscr{S}_d$ the linear space of second-order symmetric tensors on $\mathbb{R}^d$, $d = 2, 3$, or equivalently, the space $\mathbb{R}^{d\times d}_s$ of symmetric matrices of order d. We define the inner products and the corresponding norms on $\mathbb{R}^d$ and $\mathscr{S}_d$ by $u \cdot v = u_i v_i$, $\|v\| = (v \cdot v)^{1/2}$ for all $u, v \in \mathbb{R}^d$ and $\sigma : \tau = \sigma_{ij}\tau_{ij}$, $\|\tau\|_{\mathscr{S}_d} = (\tau : \tau)^{1/2}$ for all $\sigma, \tau \in \mathscr{S}_d$. The summation convention over repeated indices is used.

Let $\Omega \subset \mathbb{R}^d$ be a bounded domain with a Lipschitz boundary Γ and let n denote the outward unit normal vector to Γ. The deformation operator ε: $H^1(\Omega; \mathbb{R}^d) \to L^2(\Omega; \mathscr{S}_d)$ is defined by $\varepsilon(u) = \{\varepsilon_{ij}(u)\}$, $\varepsilon_{ij}(u) = 1/2(\partial_j u_i + \partial_i u_j)$, where $\partial_j = \partial/\partial x_j$, $i, j = 1, \ldots, d$. The spaces $L^2(\Omega; \mathbb{R}^d)$, $L^2(\Omega; \mathscr{S}_d)$ and $H^1(\Omega; \mathbb{R}^d)$ are Hilbert spaces endowed with the