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A time-accurate pseudo-wavelet scheme for parabolic and hyperbolic PDE's

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Abstract

In this paper, we propose wavelet Taylor–Galerkin schemes for parabolic and hyperbolic PDEs taking full advantage of the compression properties of wavelet basis. The discretization in time is performed before the spatial discretization by introducing high-order generalization of the standard time-stepping schemes with the help of Taylor series expansion in time step. Then, we present numerical results for a convection problem in one dimension and Gaussian translating hill problem in two dimensions. Finally, results for the two-dimensional turbulence are shown.

Keywords: Taylor–Galerkin method; Wavelets; Time-marching scheme; Navier–Stokes equation; Turbulence

1. Introduction

The application of methods based on wavelets to the numerical solution of partial differential equations (PDEs) has recently been studied both from the theoretical and the computational point of view due to its attractive feature: orthogonality, arbitrary regularity, good localization wavelet bases seem to combine the advantages of both spectral and finite element basis. Schematically, the wavelet-based methods for PDEs can be separated into three classes.

In a first class, wavelets are used, in the framework of a classical grid adaptive numerical code, to detect where the grid has to be refined or coarsened to optimally represent the

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solution. Instead of expanding the solution in terms of wavelets, the wavelet transform is used to determine the adaptive grid [7].

In a second class, multiresolution analysis and their associated scale function bases may be used as alternative bases in Galerkin methods [1,6,11]. Such methods have thus convergence properties similar to those of spectral methods, and simultaneously partial derivative operators discretize similarly as in finite difference methods. However, as these methods do not use wavelets but rather scale function as basis functions, they cannot be adaptive methods and cannot significantly reduce the number of degree of freedom in a numerical code.

The third class, the only one which uses the compression properties of wavelet bases, contain specific wavelet methods for PDEs. In the literature, many tentatives have been performed, often based on Galerkin or Petrov–Galerkin methods. Some of them take advantage of the wavelet compression of the solution [9], others instead use the wavelet compression of the operator [5]. The aim of the present paper is to introduce the wavelet Taylor–Galerkin method which has the benefit of both the properties. In the conventional numerical approach to transient problems, the accuracy gained in using the high-order spatial discretization is partially lost due to the use of low-order time discretization schemes. Here usually spatial discretization precedes the temporal discretization. On the contrary, the reversed order of discretization can lead to better time-accurate schemes with improved stability properties. The fundamental concept behind the Taylor–Galerkin approach is to incorporate more analytical information into the numerical scheme in the most direct and natural way, so that the technique may be regarded as an extension of the Obrechkoff methods to PDEs [8] for ordinary differential equations. Higher-order accurate versions of the Euler time-stepping algorithms are developed on the basis of Taylor series expansion where the time derivatives are evaluated from the governing equation. It can be generalized to any time-stepping scheme based on Taylor series expansion.

The nonlinear or variable coefficient term is evaluated by a pseudowavelet technique. Spatial approximation can be made by using different wavelet bases such as orthogonal Daubechies wavelets [3], biorthogonal spline wavelets [2], interpolates [4], etc. Our method works with any of these basis functions. In this paper, we demonstrate our method using Daubechies compactly supported wavelets.

2. Wavelet Taylor–Galerkin method (WTGM) for evolutionary problems

In the following, we give a brief introduction to wavelets and the notation used. We first deal with one-dimensional wavelets and then consider two variants for its generalization to the multivariate case. Finally, we describe how compression properties wavelets can be used in WTGM scheme for convection problem in one dimension and multidimensional problems.

2.1. Univariate wavelets

The class of compactly supported wavelet bases was introduced by Daubechies [3] in 1988. They are an orthonormal basis for functions in $L^2(R)$. A “Wavelet System” consists