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Asymptotic stability of rarefaction wave of the Cauchy problem for viscous conservation laws

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Abstract

This paper is concerned with the asymptotic stability of rarefaction waves for the two-dimensional steady isentropic irrotational flow with artificial viscosity. We prove that if the initial value is close to a constant state and the corresponding inviscid hyperbolic system admits a weak rarefaction wave, then the solution tends to this rarefaction wave as $t \to \infty$.

MSC: 35L45; 35L65

Keywords: Asymptotic stability; Viscous conservation laws; Rarefaction wave; Perturbation

1. Introduction

In fluid dynamics, the inviscid hyperbolic system is expected to describe the large scale structures of general viscous flows which are governed by viscous conservation laws. The studies on the general viscous conservation laws have been the center of the continuum mechanics. One of the major issues is to obtain the global existence and the large time behavior of solutions to viscous systems. In the past decades, there has been a great success achieved in this field. Il'in and Oleinik [1] first established the one-dimensional stability of rarefaction waves for scalar viscous conservation laws based on a maximum principle. After then, Matsumura and Nishihara [4] proved the asymptotic stability of the weak rarefaction wave in the case of p-system with viscosity, later in [5], they improved the previous

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result by removing the smallness condition of the initial data and weakness of the rarefaction wave. Xin [8,9] considered the asymptotic behavior toward weak rarefaction waves of the solution to a general 2×2 hyperbolic conservation laws with positive viscosity. For two-dimensional scalar viscous conservation laws, Xin [10] succeed in showing that there exists a global solution in time which tends to the one-dimensional rarefaction wave. Nishikawa and Nishihara [6] obtained the stability of the planar rarefaction wave without the smallness conditions. What they considered was the asymptotic behavior of solution to one-dimensional systems or two-dimensional scalar equation. Therefore, it is natural to study two-dimensional steady isentropic irrotational viscous flow.

First, let us recall some properties of the two-dimensional steady inviscid isentropic irrotational flow

$$\begin{cases} v_x - u_y = 0, \\ (\rho u)_x + (\rho v)_y = 0, \end{cases} \tag{1.1}$$

where u and v are the velocities of the flow in the direction of x- and y- axis respectively, ρ is the density which is determined by the Bernoulli relation

$$\frac{1}{2}(u^2 + v^2) + \frac{a^2}{\gamma - 1} = c, \tag{1.2}$$

where c is a positive constant, $\gamma \in (1, 2]$ is a constant representing adiabatic exponent, from the pressure density relation $p = \rho^\gamma$, the sound speed $a(\rho)$ is defined by $a^2(\rho) = \mathrm{d}p/\mathrm{d}\rho$.

System (1.1) is hyperbolic and genuinely nonlinear if the x-direction is regarded as the time direction. Direct computation implies that system (1.1) has two eigenvalues

$$\lambda_1 = -uv + a\sqrt{u^2 + v^2 - a^2}/a^2 - u^2,$$
$$\lambda_2 = -uv - a\sqrt{u^2 + v^2 - a^2}/a^2 - u^2,$$

and the corresponding right eigenvectors are $r_i = (1, -\lambda_i)^{\mathrm{T}}$ $(i = 1, 2)$. Moreover,

$$\nabla\lambda_i(v, u) \cdot r_i(v, u) = \frac{(\gamma + 1)(\lambda_i^2 + 1)\left(-u\sqrt{u^2 + v^2 - a^2} \pm av\right)}{2(a^2 - u^2)\sqrt{u^2 + v^2 - a^2}} \triangleq d(v, u) \neq 0, \tag{1.3}$$

where $i = 1, 2$ is corresponding to $+, -$. Hence system (1.1) is genuinely nonlinear. For simplicity, we rescale $r_i = [1/d(v, u)](1, -\lambda_i)^{\mathrm{T}}$ so that (1.3) becomes

$$\nabla\lambda_i(v, u) \cdot r_i(v, u) \equiv 1. \tag{1.4}$$

First of all, what we are concerned is the addition of viscosity of system (1.1). Physically speaking, the viscous flow corresponding to (1.1) can be described by the following system:

$$\begin{cases} (\rho u)_x + (\rho v)_y = 0, \\ (p + \rho u^2)_x + (\rho v u)_y = (\mu' + \frac{4}{3}\mu)(u_{xx} + u_{yy}), \\ (\rho v u)_x + (p + \rho u^2)_y = (\mu' + \frac{4}{3}\mu)(v_{xx} + v_{yy}), \end{cases} \tag{1.5}$$

where μ and μ' represent the first and the second viscous coefficient respectively. This mathematical model can be directly derived from Navier–Stokes equations. Although so far