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Hemivariational inequality for viscoelastic contact problem with slip-dependent friction ☆

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Abstract

In this paper we consider a class of hyperbolic hemivariational inequalities modeling the frictional contact between a viscoelastic body and a rigid foundation. The friction law is described by a multi-valued subdifferential relation which includes the slip displacement dependent friction and the Tresca models. We establish the existence of weak solutions to the problem by using a surjectivity result for pseudomonotone mappings.

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Keywords: Hemivariational inequality; Friction; Viscoelasticity; Slip; Subdifferential; Contact problem; Nonconvex

1. Introduction

In this paper we study the existence of solutions to the hemivariational inequality modeling the dynamic process of frictional contact between a deformable body and a foundation. The viscoelastic body is supposed to satisfy the constitutive relationship of Kelvin–Voigt type with a linear elasticity operator and a nonlinear viscosity operator. We consider the time-dependent Tresca model assuming that the normal stress on the contact surface is

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prescribed. The friction is modeled by a subdifferential boundary condition which in particular covers the case when the tangential shear on the contact surface is a possibly multivalued function of the tangential velocity. The multivalued boundary condition has a nonmonotone character since this condition comes out from the nonconvex superpotential. This implies that a convex analysis approach to the contact problem is not possible and we are naturally lead to hemivariational inequality formulation.

Hemivariational inequalities were introduced by Panagiotopoulos [21,22] to describe various mechanical problems involving the nonconvex energy functionals. The derivation of hemivariational inequality is based on the notion of the generalized gradient of Clarke. In the case of convex superpotentials the hemivariational inequalities reduce to variational inequalities (see [7]). In the elliptic and parabolic cases, several types of hemivariational inequalities have been studied and they proved to be efficient for solving many mechanical problems (cf. [18,17]). The existence results for hyperbolic hemivariational inequalities have been delivered only recently by Goeleven et al. [8], Haslinger et al. [9], Migórski [15], who treated problems with a subdifferential depending on the first-order time derivative of the unknown function. Panagiotopoulos and Pop [23], Haslinger et al. [9], Ochal [19,20] and Migórski [16] have studied hemivariational inequalities with a multivalued term depending on the unknown function and not on its derivative. The contact problems for the viscoelastic models have attracted many attention in recent years (see e.g. [13,14,10,16] and the references therein).

The goal of this paper is to provide an existence result for the dynamic hemivariational inequality modeling the contact phenomenon. We extend the approach used in Migórski [16] and Ochal [19,20] to the case when the subdifferential term depends on both the unknown function and its first-order time derivative. This case covers in particular the mathematical formulation of the contact problem with the friction coefficient depending on the slip displacement (cf. [24,10–12], where the friction coefficient was supposed to vary with the tangential displacement). On the other hand our method allows to treat the multidimensional and multivalued friction laws of zig-zag type studied earlier by Panagiotopoulos [22].

In order to cope with the multivalued nonlinearities in the problem, we assume that the superpotential is regular in the sense of Clarke. This allows to give an equivalent formulation of the hemivariational inequality under consideration in the form of evolution inclusion of second order. Next, we transform the latter, assuming a regular initial data, into an evolution inclusion of first order and apply a surjectivity result for L-pseudomonotone and coercive operators (cf. [6]). Then, by using a density argument we remove the restriction on the regularity of the initial condition and prove the result in a general case. We remark that our framework incorporates a nonlinear viscosity operator satisfying a pseudomonotonicity condition, a positive and in general noncoercive elasticity operator, and the subdifferential of a general nonsmooth superpotential.

The content of this paper is as follows. In Section 2 we recall some notation and present some auxiliary material. In Section 3 we state the mechanical problem and describe contact boundary conditions. We derive the variational formulation of the model and state the hypotheses in Section 4. The formulation of the evolution inclusion of second order is given in Section 5. The results on the existence of the weak solution to the hemivariational inequality as well as to the contact problem are delivered in Section 6. Section 7 contains the proofs of some auxiliary results.