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Nash bargaining solution under externalities[☆]

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HIGHLIGHTS

- We define a Nash bargaining solution (NBS) for partition function games.
- We define a bargaining game where the rejecter of a proposal stochastically exits.
- We show that the NBS is supported by any efficient stationary equilibrium.
- We provide a necessary and sufficient condition for such an equilibrium to exist.

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ABSTRACT

We define a Nash bargaining solution (NBS) of partition function games. Based on a partition function game, we define an extensive game, which is a propose–respond sequential bargaining game where the rejecter of a proposal exits from the game with some positive probability. We show that the NBS is supported as the expected payoff profile of any stationary subgame perfect equilibrium (SSPE) of the extensive game such that in any subgame, a coalition of all active players forms immediately. We provide a necessary and sufficient condition for such an SSPE to exist. Moreover, we consider extensions to the cases of nontransferable utilities, time discounting and multiple-coalition formation.

1. Introduction

When we regard coalition formation with externalities as a bargaining problem, it is a natural scenario that if a player disagrees, negotiations would be terminated and every player would stand alone. In this scenario, each player's threat payoff is her payoff when every player stands alone. Formally, in a partition function game (N, V), player i's disagreement results in the coalition structure $\{\{j\} \mid j \in N\}$, her threat payoff is $V(\{i\}, \{\{j\} \mid j \in N\})$, and the threat payoff profile is $(V(\{i\}, \{\{j\} \mid j \in N\}))_{i \in N}$.

Another plausible scenario is that if a player disagrees (or deviates from the agreement), the other players would cooperate and she would be isolated. In this scenario, each player's threat payoff is her payoff when she is isolated. Formally, player i's disagreement results in the coalition structure $\{\{i\}, N \setminus \{i\}\}$, her threat payoff is $V(\{i\}, \{\{i\}, N \setminus \{i\}\})$, and the threat payoff profile is $(V(\{i\}, \{\{i\}, N \setminus \{i\}\}))_{i \in N}$.

In each scenario, we define the Nash bargaining solution (NBS) of a partition function game: *fine NBS (fNBS)* and *coarse NBS (cNBS)*. The fNBS (cNBS) of partition function game (N, V) is the NBS of the bargaining problem such that players bargain over the worth $V(N, \{N\})$ under the threat payoff profile $(V(\{i\}, \{\{j\} \mid j \in N\}))_{i \in N}$ $((V(\{i\}, \{\{i\}, N \setminus \{i\}\}))_{i \in N})$.[1] If there is no externality, the difference between the two scenarios is irrelevant, and the fNBS coincides with the cNBS. However, if externalities between coalitions exist, a bargaining outcome would depend on which scenario each player imagines in negotiations. The cNBS is newly defined by this study.

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1 If positive externalities are strong, the cNBS of a partition function game does not exist because $\sum_{i \in N} V(\{i\}, \{\{i\}, N \setminus \{i\}\}) > V(N, \{N\})$.

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Conceptually, we propose distinguishing threat payoff profiles from disagreement points. A disagreement point is a payoff allocation when some player disagrees, and it may depend on the identity of the player who disagrees. Each player's threat payoff is her payoff in the disagreement point that she causes. Because the value of an outside option for each player is her threat payoff, players' bargaining positions are determined not according to the disagreement points, but according to the threat payoff profile. Needless to say, if the disagreement points do not depend on the identity of the player disagreeing, the disagreement point caused by each player coincides with the threat payoff profile. Because each player's threat payoff is the payoff that she would obtain if she disagreed, the threat payoffs do not have to be consistent. Thus, the threat payoffs $(V(\{i\}, \{\{i\}, N \setminus \{i\}\}), i \in N)$ for the cNBS can be inconsistent: coalition structures $\{\{i\}, N \setminus \{i\}\}$ and $\{\{j\}, N \setminus \{j\}\}$ may not coexist for some distinct $i, j \in N$.

According to Gomes (2005), the fNBS of a partition function game is supported as the expected payoff profile of a stationary subgame perfect equilibrium (SSPE) of an extensive game.[2] However, the cNBS has no noncooperative foundation.

This study provides a noncooperative foundation for the cNBS of partition function games. Based on a partition function game, we define an extensive game, which is a propose–respond sequential bargaining game where the rejecter of a proposal exits from the game with a positive probability (*rejecter-exit partial breakdown*). We show that the expected payoff profile of any full-coalition SSPE (SSPE such that in any subgame, the coalition of all active players forms immediately) coincides with the cNBS. We also provide a necessary and sufficient condition for a full-coalition SSPE to exist. Moreover, we consider extensions to the cases of nontransferable utilities, time discounting and multiple-coalition formation.

The fNBS and cNBS are defined by the following two-step approach: first, define a characteristic function game based on the partition function game; second, let the NBS of the characteristic function game be the NBS of the partition function game. For the fNBS (cNBS), in the first step, a characteristic function game (N, v) based on partition function game (N, V) is defined as follows: for any $S \in 2^N \setminus \{\emptyset\}$, $v(S) = V(S, \{S\} \cup \pi)$, where π is the finest (coarsest) partition of $N \setminus S$, i.e., $\pi = \{\{i\} \mid i \in N \setminus S\}$ ($\pi = \{N \setminus S \mid i \in N \setminus S\}$).[3] We refer to this as the *fine way* (*coarse way*). The approach of defining characteristic function games from partition function games is used to define the Shapley value and the core of the partition function games. de Clippel and Serrano (2008) and McQuillin (2009) axiomatize the Shapley values of partition function games defined by the fine and coarse ways, and refer to them as the *externality-free Shapley value* and the *extended, generalized Shapley value*, respectively. They point out that the externality-free Shapley value and the extended, generalized Shapley value are supported as equilibrium payoff profiles in the extensive games of Hart and Mas-Colell (1996) and Gul (1989), respectively. Hafalir (2007) defines the cores of partition function games by fine and coarse ways, and refers to them as the *core with singleton expectations* and the *core with merging expectations*, respectively.

In this study, the disagreement situation may depend on who disagrees (*non-anonymous disagreement*). Several studies consider bargaining problems with non-anonymous disagreements. Kıbrıs and Tapkı (2010) investigate bargaining problems with non-anonymous disagreements in a cooperative approach. In Kıbrıs and Tapkı (2010), each player's disagreement determines an entire allocation in disagreement. On the other hand, in the cNBS in this study, player i's disagreement determines her payoff and the worth of the coalition of the other players, but does not determine an allocation among the other players, which does not matter in defining the cNBS. Corominas-Bosch (2000) considers a noncooperative bargaining game with non-anonymous disagreements. However, the model only includes two players, and thus does not consider coalition formation.

A feature of our model is to consider a partial breakdown instead of discounting. The partial breakdown represents a situation where some of the players exit during negotiations. A dropout can occur if a player dies because of differences in lifetimes or if she chooses outside opportunities. Furthermore, members might compulsorily leave some players out of the negotiations. Partial breakdowns capture the aspects of negotiations that discounting cannot capture.

This study examines the rejecter-exit partial breakdown, in which if players fail to agree, the rejecter exits from the game with a certain probability. One rationale for this partial breakdown is that the rejection of a proposal leads to the proposer becoming hostile toward the rejecter, and the rejecter is forced to exit from the negotiation. After player i exits from the game by partial breakdown in the first round, the other players form coalition $N \setminus \{i\}$ in the full-coalition SSPE, coalition structure $\{\{i\}, N \setminus \{i\}\}$ is realized, and player i obtains payoff $V(\{i\}, \{\{i\}, N \setminus \{i\}\})$. This underlies the fact that the expected payoff profile of any full-coalition SSPE in the limit is equal to the cNBS.

Miyakawa (2008), Calvo (2008), and Hart and Mas-Colell (1996) consider partial breakdowns. In Miyakawa (2008), a responder is randomly selected and exits from the game. In Calvo (2008), a player is randomly selected and exits from the game. In Hart and Mas-Colell (1996), the proposer exits from the game.[4] While the Shapley value is noncooperatively supported under the proposer-exit partial breakdown in Hart and Mas-Colell (1996), the Nash bargaining solution is noncooperatively supported under the rejecter-exit partial breakdown in the present study. On the one hand, the above-mentioned studies on bargaining with partial breakdowns do not consider externalities. On the other hand, studies on bargaining with externalities do not consider partial breakdowns (e.g., Bloch, 1996; Ray and Vohra, 1999).

The remainder of the paper is organized as follows. Section 2 defines the NBSs of partition function games. Section 3 presents an extensive game based on a partition function game. Section 4 shows that the cNBS is supported by the expected payoff profile of any full-coalition SSPE. Section 5 provides a necessary and sufficient condition for the existence of a full-coalition SSPE. Section 6 considers extensions to the cases of nontransferable utilities, time discounting and multiple-coalition formation. Section 7 concludes the paper. The proofs of all theorems are given in Appendix.

2. Nash bargaining solution

For any sets X and Y, let Y^X be the set of functions from X to Y. For any function f and any x in the domain of f, let f_x be the image of x under f, i.e., $f_x := f(x)$. For any set A, a *partition* of A is π such that $\pi \not\ni \emptyset$, $S \cap T = \emptyset$ for any distinct $S, T \in \pi$ and $\bigcup \pi = A$.[5,6]

2 Okada (2010) investigates extensive games based on strategic games. He shows that the fNBS suitably defined in a strategic game is noncooperatively supported.

3 If $S \neq N$, $\{N \setminus S \mid i \in N \setminus S\} = \{N \setminus S\}$; otherwise, $\{N \setminus S \mid i \in N \setminus S\} = \emptyset$.

4 Hart and Mas-Colell (1996) also generalize the bargaining procedure as follows: a player is selected with a probability and exits from the game; this probability may depend on the identity of the proposer. However, they do not depend on the identity of the rejecter. Therefore, the present study's procedure is not a special case of the generalized procedure of Hart and Mas-Colell (1996).

5 In this study, for any set $\mathcal{A}$, $\bigcup \mathcal{A} := \{a \mid \exists A \in \mathcal{A}\ (a \in A)\}$. Some authors denote this by $\bigcup_{A \in \mathcal{A}} A$.

6 According to this definition, the empty set is a unique partition of the empty set.