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Bargaining order and delays in multilateral bargaining with heterogeneous sellers

Shubhro Sarkar

Indira Gandhi Institute of Development Research, Gen. A.K. Vaidya Marg, Goregaon East, Mumbai, 400065, India

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ABSTRACT

We consider a complete-information multilateral bargaining game in which a single buyer negotiates with two heterogeneous sellers selling perfect complementary units. While bilateral negotiations take place through a sequence of offers and counteroffers, the bargaining order is exogenously given. We solve for the conditions under which (a) the buyer prefers to negotiate with the lower-valuation seller first and (b) efficient (inefficient) outcomes emerge for the two bargaining orders. We find that the buyer prefers to negotiate with the lower-valuation seller first whenever the players are relatively impatient or the sellers are sufficiently heterogeneous. We show that there exists a unique efficient outcome when the buyer negotiates first with the lower-valuation seller and the sellers are sufficiently heterogeneous; however, significant delay in reaching agreements may arise when they are not. In case the buyer bargains with the higher-valuation seller first, an inefficient outcome is shown to exist even when players are extremely impatient.

1. Introduction

It has been well-documented that multilateral bargaining games comprising patient homogeneous sellers selling perfect complementary units, suffer from a hold-up problem, when each seller endeavors to reach agreements later, with the hope of securing a larger share of the surplus (Cai, 2000). This leads to inefficient delays. The purpose of this paper is to analyze the sequence in which a buyer prefers to negotiate with sellers with different valuations for their objects and to solve for the conditions under which (in)efficient outcomes exist in such bargaining games.

There are several examples of multilateral negotiations where a single buyer has to negotiate with multiple heterogeneous sellers. These include an industrialist bargaining with several farmers in order to assemble plots of land for a project; a manufacturer negotiating with a group of upstream suppliers; and a manager bargaining with two different unions in order to end a strike. In each of these examples, it is possible that the sellers have different valuations for their objects. In the land assembly problem for example, sellers could be expected to have different valuations for their land, even if the plots are contiguous and similar in size, when they have different endowments of skill and capital or have varying access to alternative methods of earning a livelihood (Ghatak and Ghosh, 2011).

To examine whether the hold-up problem is mitigated or exacerbated in the presence of heterogeneous sellers, we consider a multilateral bargaining problem with one buyer and two sellers. We assume that each seller owns a single object, and that the objects are *perfect complementary* to the buyer, such that she realizes the value of a project (M) only when she reaches an agreement with both the sellers. The two sellers have different valuations for their objects. For analytical tractability, we normalize the value of the lower-valuation seller (V_1) to zero and assume the value of the higher-valuation seller (V_2) to be strictly positive. We assume that bargaining proceeds through an exogenously determined sequence, with the buyer negotiating with each of the sellers in alternate rounds. Each round of bargaining potentially consists of two periods. In the first period the buyer makes an offer to the seller. If the offer is rejected, the seller makes a counter-offer to the buyer in the second period, which the buyer then accepts or rejects. Both the offer and the counter-offer specify the compensation (price) to be paid to the corresponding seller. If either the offer or the counter-offer is accepted, the buyer pays the seller the negotiated price and the seller leaves the game forever. Once an agreement is reached, the buyer proceeds to negotiate with the remaining seller through an infinite horizon, alternate offer bargaining game (à la Rubinstein, 1982). If on the other hand, both the offer and counter-offer are rejected, the buyer moves on to the next round to bargain with the other seller through an identical sequence of offers and counter-offers. Clearly there are two possible bargaining orders: in the first,

E-mail address: shubhro@igidr.ac.in.

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the buyer negotiates with the lower-valuation seller first, and in the second, with the higher-valuation seller.

The key issue in such problems lies in the circumstances under which different players choose to hold out, which when combined with a bargaining order, results in inefficient delays. Following the literature on homogeneous sellers, the incentive to hold-up for all the agents can be predicted to grow as the discount factor approaches one. For sufficiently patient homogeneous players, it has been shown that two of the three players choose to play "hold-up" strategies, and place the remaining player into a disadvantageous bargaining position (Cai, 2000). However with heterogeneous sellers, as the valuation of the higher-valuation seller rises, it is not immediately obvious when this seller would choose to play such "hold-up" strategies. If the higher-valuation seller is the first to reach an agreement, he would seek a larger compensation for giving up a land of higher value; if on the other hand, the seller chooses to hold out, the available compensation is rising in his valuation. We are therefore interested in answering the following questions:

(a) under what conditions does the buyer prefer to bargain with the lower-valuation (higher-valuation) seller first?
(b) given a bargaining order, what are the conditions which lead to efficient (inefficient) outcomes?

Using *stationary* subgame perfect equilibrium (SSPE) as the solution concept, we show that there exist several equilibria, of which two are significant. In the first equilibrium (E3), negotiations between the buyer and the higher-valuation seller hold out, while in the second (E5), the buyer is able to successfully negotiate the first deal with the higher-valuation seller only. Negotiations between a buyer and a seller hold out when these players adopt "hold-out" strategies and the remaining seller plays an "accommodative" strategy. We find that the buyer prefers to negotiate with the lower-valuation seller first, whenever $K < \frac{1+\delta^7-\delta^2-\delta^4}{1+\delta^3-\delta-\delta^4}$ or $K > \frac{\delta}{1+\delta}$, where $K = V_2/M$ and δ denotes the discount factor.[1] These conditions entail that either the players have to be relatively impatient or that the sellers have to be sufficiently heterogeneous. The intuition behind this result is that for $K > \frac{\delta}{1+\delta}$, the buyer is unable to make a deal with the higher-valuation seller first, since the surplus available in the first transaction is negative for such parameter values.[2] On the other hand for $K \le \frac{\delta}{1+\delta}$ and $K < \frac{1+\delta^7-\delta^2-\delta^4}{1+\delta^3-\delta-\delta^4}$, the buyer is able to successfully negotiate with the lower-valuation seller first, in case there is a hold-out with the higher-valuation seller (E3); in the case where there is no hold out, the buyer receives less surplus in the last negotiation against the higher-valuation seller, but is compensated by the lower payment she makes to the lower-valuation seller in the first negotiation.

We answer question (b) in Proposition 9, in which, inter alia we show (i) that for $K > \frac{\delta}{1+\delta}$ there exists a unique efficient (inefficient) equilibrium outcome when the buyer bargains with the lower (higher) valuation seller first, and (ii) that for $K \le \frac{\delta}{1+\delta}$ and $K \ge \frac{1+\delta^7-\delta^2-\delta^4}{1+\delta^3-\delta-\delta^4}$, there exist multiple equilibria,[3] of which one leads to inefficient delay when the buyer negotiates with the lower-valuation seller first (E5). The second part of this result is similar to that of Cai (2000) who studies a multilateral bargaining model of complete information, in which one buyer negotiates with two *homogeneous sellers* selling perfect complementary units. He shows that when players are sufficiently patient, inefficient delays may emerge as an equilibrium outcome. However the indeterminacy of equilibrium outcomes disappears once the sellers are *sufficiently heterogeneous* as shown in the first part of our result. We show that it is possible to generate efficient outcomes for such parameter values provided the buyer negotiates with the lower-valuation seller first. On the other hand, if the buyer starts by negotiating with the higher-valuation seller, there will be inefficient delays even if the players are extremely impatient. Given that there are instances where negotiations have failed when the participants have deemed the outcome to be unfair, we calculate the Gini coefficient for the equilibria E3 and E5. We find that in the first equilibrium it is a constant, and that it increases with K in the second equilibrium.

Finally, to check the relevance of our results for alternative bargaining protocols, we allow the buyer to either negotiate with the same seller for $k > 1$ rounds, or to negotiate till an agreement is reached. For perfectly patient players we find that the preferred bargaining order for the buyer does not change when she is allowed to negotiate with the same seller for a finite number of rounds if $K > 1/2$. The buyer prefers to bargain with the lower-valuation seller first. With $K \le 1/2$ there are multiple equilibria, in one of which the buyer prefers to negotiate with the higher-valuation seller first. In case the buyer is allowed to bargain with the same seller for an infinite number of rounds, she then prefers to begin with seller 1.

While Coase (1960) provides the most famous example of the holdout problem where he describes a railroad trying to acquire plots of land from several farmers, Eckart (1985) and Asami (1988) were the first to offer game theoretic arguments to study such problems. Holdout-related inefficient delays are also salient in Cai (2003) and Menezes and Pitchford (2004). While the former studies a model similar to Cai (2000) with *contingent* contracts and shows that there are multiple Markov equilibria, the latter uses a two-seller framework with cash-offer contracts, and allows the buyer to negotiate with both the sellers at any given date. In a recent paper Chowdhury and Sengupta (2012) broadly analyze the conditions which lead to a (in)significant holdout problem and find that the problem is largely resolved when either the bargaining protocol is transparent and the buyer has a positive outside option or the marginal contribution of the last seller is not too large. However, it continues to be severe whenever the buyer has no outside option or when the bargaining protocol is secret. Strategic holdout has also been shown to pose a serious problem to R&D development, when licenses have to be obtained from multiple patentees (Shapiro, 2001).

In addition to the seller holdout problem our paper is related to the literature on optimal negotiation sequence. Strategic sequencing by a buyer negotiating with two sellers owning complementary units is the focus of a paper by Krasteva and Yildirim (2012). They assume that the buyer bargains with each seller individually and sequentially through a one-shot random-proposer bargaining protocol, and that the bargaining power of a seller is given by the probability with which he gets to make a take-it-or-leave-it offer. A paper closely related to ours is Xiao (2015), who examines, inter alia, the buyer's preference over bargaining orders in an infinite-horizon complete information multilateral bargaining game with asymmetric sellers. He assumes that the bargaining order is endogenously determined by the buyer and shows that the buyer chooses to negotiate in order of increasing size when the sellers are of sufficiently different sizes. In contrast, we chose to carry out our analysis through an exogenously given (symmetric) bargaining protocol.[4] This allowed us to answer

[1] Question (a) is answered in Propositions 10 and 11 in Section 4.

[2] See proof of Proposition 8.

[3] The parameter space which satisfies these conditions is represented by the region ABGE in Fig. 2.

[4] By symmetric bargaining protocol, we mean that the buyer negotiates with the two sellers over the same number of rounds, before she switches to the other seller.