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Identifying reaction functions in differential oligopoly games ☆

Luca Lambertini [a,b,*], Andrea Mantovani [a,c]

[a] Department of Economics, University of Balogna, Strada Maggiore 45, I-40125 Balogna, Italy
[b] ENCORE, Faculty of Economics & Econometrics, University of Amsterdam, Roetersstraat 11, WB1018 Amsterdam, The Netherlands
[c] CORE, Université Catholique de Louvain, 34 Voie du Roman Pays, B-1348, Louvain-la-Neuve, Belgium

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Abstract

We investigate the issue of strategic substitutability/complementarity and define the concepts of best reply function and dominant strategy in deterministic differential games. We prove that if a player's Hamiltonian is additively separable w.r.t. controls, then players follow dominant strategies at every instant. Otherwise, if the Hamiltonian is not additively separable w.r.t. controls, instantaneous best replies can be properly characterised. However, under additive separability, we show that strategic interaction via best replies can still be characterised at the steady state. Illustrative examples are Ramsey and Solow's growth models, reformulated as oligopoly games, and a Cournot differential game with sticky price.

Keywords: Complementarity; Substitutability; Differential games

JEL classification: C73; D43; D92; L13

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* Corresponding author. Department of Economics, University of Balogna, Strada Maggiore 45, I-40125 Balogna, Italy.
E-mail addresses: lamberti@spbo.unibo.it (L. Lambertini), mantovani@core.ucl.ac.be (A. Mantovani).

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1. Introduction

The issue of super-/submodularity has been investigated mostly in static games, and refers to the slope of reaction functions in the (stage) game, as initially pointed out by Bulow et al. (1985). Recently, the concept of economic complementarity has emerged as a leading theme of research. The analysis has been focused on games with strategic complementarities and their use in industrial economics (Vives, 1990; Milgrom and Roberts, 1990; Amir, 1996b) and in comparative statics analysis (Milgrom and Shannon, 1994). Echenique (2004) proves that an extensive form game with strategic complementarity yields a set of subgame perfect Nash equilibria which is a nonempty and complete lattice. Markovian equilibria of stochastic games are investigated by Amir (1996a) and Curtat (1996) in discrete time stochastic games with complementarities.

A potential development of this discussion consists in investigating whether the same properties can be reconstructed in a differential game, and to what extent.[1] The static game approach obviously rules out *a priori* the possibility of *intertemporal* strategic substitutability/complementarity. On the contrary, by using a differential game approach we can enlarge the class of games showing strategic complementarities by considering firms' interaction *at the steady state*.

To the best of our knowledge, only Jun and Vives (2004) have considered intertemporal strategic complementarity/substitutability in a differential oligopoly game unraveling over continuous time. They compare steady states of open-loop and stable closed-loop equilibria in a general symmetric differential duopoly model with adjustment costs, as in Reynolds (1987) and Driskill and McCafferty (1989). One of the most interesting result appears in the "mixed" case of price competition and production adjustment costs: the strategic complementarity of the static game turns into an intertemporal strategic substitutability, given that, from the standpoint of any given firm, a price cut today makes the rival smaller in the future by raising its short-run marginal cost.[2]

A related issue is that of conjectural variations. Dockner (1992) shows that any closed-loop (i.e., subgame perfect) equilibrium coincides with a conjectural variations equilibrium. Using a Cournot model with linear demand and quadratic production costs, he proves that the dynamic conjectural variations consistent with the closed-loop equilibrium are negative constants which, depending upon the level of the discount rate, vary between zero and the consistent conjectures characterising the static version of the same game.

The aim of the present paper is to identify best reply functions in differential oligopoly games. While in static games the best replies (or reaction functions) obtaining from first order conditions characterise the existence, unicity/multiplicity and stability of equilibria, in differential games the instantaneous best replies only define the optimal behaviour of any single player at each particular instant. As we shall point out in the remainder of the paper, a generic player's first order conditions in a differential game may not depend upon the rival's controls. If they do, then we can define the instantaneous best replies. If they don't, then we say that the solution of a first order condition on any given player's control yields an instantaneous dominant strategy, by analogy with what we observe in a static game where best replies are flat. In such a case, however, it is

[1] For an overview of differential game theory and applications, see Başar and Olsder (1982, 1995[2]), Mehlmann (1988) and Dockner et al. (2000). For differential oligopoly games, see Cellini and Lambertini (2003).

[2] Very little has been done thus far concerning the empirics of such issues. Zulehner (2003) tests the presence of dynamic strategic interaction in the dynamic random access memory (DRAM) industry. She estimates a dynamic oligopoly model incorporating learning-by-doing and spillovers, showing that firms take into account the intertemporal strategic effect of their contemporaneous output decisions on their rivals' future output decision. Jarmin (1994) raises the same question by looking at the early rayon industry and found empirical evidence of dynamic strategic behavior.