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The stochastic evolution of rumors within a population

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A B S T R A C T

We study rumor propagation process with incubation and constant immigration. We take into account a deterministic rumor spreading model and demonstrate the persistence of a rumor when the basic reproduction number is greater than one. Due to the presence of a randomness in the influence that the incubators exert on ignorants, we extrapolate the deterministic rumor model to a stochastic one by using a stochastic coefficient for the term representing the latter influence within the system. The existence and boundedness of both local and global solutions are demonstrated. We prove the uniqueness of these solutions. Conditions of extinction is also established. We perform numerical simulations to verify our stochastic model. The present work can assist decision takers in the analysis of the dynamical evolution of rumors in a given society as well as in the study of information dissemination strategies.

1. Introduction

Rumor forms part of our daily lives, and its spread has a considerable influence on human lives. Hayakawa (2002) defines rumor as a kind of social phenomenon that a remark spreads on a large scale in a short time through chains of communications and runs through the whole evolutionary history of mankind. Basically speaking, a rumor is a doubtful statement in motion. Its origin is most of the time unknown since it is easily spread by words of mouth. Along its spread, the rumor is constantly being modified with information that is of interest of people. Based on ambiguous sources, rumor has a major impact on individuals and institutions. In many situations, rumor is known to not only affect people psychologically, inducing anxiety and fears, damaging reputations and destroying morale but also to harm the economy. Nearly as old as human history, rumor has become omnipresent with the ever growing communication technology. Different rumors have different effects, but at the end of the day, everybody's life is affected. Taking all these into consideration and the fact that spread of rumors does not choose any particular age group, it is essential for us to study the propagation of rumors and analyze the parameters that make rumors persist in our modern society.

2. Some existing models of dynamics of rumor transmission

It is imperative for decision takers to express the trend in the expansion of the different rumor transmission classes. They should be able to distinguish how each rumor class responds to rumor propagation in order to avoid the above-mentioned problems.

The spread of rumors and that of epidemic infections have much in common. Epidemiological models involve analyzing the spread of a disease. They divide the population into different classes (commonly named, susceptible, infected and recovered), reflecting the health status of each individual. At each successful contact, an individual passes from one class to another. Infectious, like the spreaders in the rumor propagation model, infect the susceptibles or notify the ignorants, causing the latter to shift from "susceptible" or "ignorant" class to the "infected" or "spreader" class. We therefore make use of epidemiological models to study the spread of rumors both deterministically and stochastically.

A classical model of rumor propagation was studied in Daley and Kendall' (1965). The latter authors divided the population into three classes, namely, ignorant, spreader and stifler. However, there are some issues pertaining to their study. The model does not allow any inflow or outflow to the ignorant class or from the other classes. They also assumed that an ignorant should absolutely become a spreader upon hearing a rumor, which is completely false in real life.

Thompson et al. (2003) study the propagation of rumors based on two groups of people, specifically, passive and active. The passive is defined as those who have less contacts in a day and hence are not strongly interested in spreading rumors, while the active is completely the opposite. Different cases of rumors have been analyzed: frivolous, boring, interesting and unbelievable. The unbelievable rumor has been concluded to be the best case of rumor propagation. However, the study did not take into account the fact that a spreader might lose interest in a rumor and thus

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decide to stop spreading it. In other words, the latter becomes stifler without making any contact with either another spreader or stifler. Moreover, there is no analytic representation of the rumor endemic equilibrium.

According to Zhao and Wang (2013), the transmission of rumors is not only due to contacts between different classes, but also due to a medium such as website and social networks. Nevertheless, the system does not consider inflow or outflow to or from the classes. The individuals only transit from one rumor class to another. Furthermore, the model is based on rough conditions as they assumed the contact number and followers of every person to be the same. In Zhao and Wang (2014), the authors shows how the government measures affect rumor propagation. These include issuing actual messages through the medium and taking legal actions against the spreaders.

In Fedewa et al. (2013), the authors use the discrete hypergeometric distribution to model rumor propagation based on a sampling without replacement. In their study, they considered that a person may become stifler in three ways: contact between (i) two spreaders, (ii) a spreader and a stifler, and (iii) a spreader and an ignorant. However, they did not take into account that a spreader may automatically lose interest and become stifler. Moreover, no parameter is used to indicate the rate of change and proportion of an individual changing his rumor class. In addition, the three different ways of becoming stiflers are considered as three different models instead as a whole one to reflect the reality.

Regarding the process on a heterogeneous mixing population, de Arruda et al. (2004) set up a system of differential equations that describes the time evolution of the probability that an individual is in each state. However, no inflow and outflow are considered within the model. The whole population is partitioned into three different classes: ignorant, spreader and stifler. Moreover, no contact between two spreaders is taken into account. In that model, the spreader cannot transit to the stifler class directly. That is, a spreader is assumed to have permanent interest in propagating a rumor.

A general stochastic rumor model for the spread of rumors is introduced in Nekovee et al. (2007) and the authors use mean-field equations to describe the dynamics of the model on complex social networks. Another study due to Moreno et al. (2004), derives mean-field equations to characterize the dynamics of a rumor process that takes place on top of complex heterogeneous networks. Nevertheless, they both do not consider the inflow and outflow within the system. They assume the network to be static, in other words, a time-independent network topology. Moreover in Moreno et al. (2004), a spreader is assumed to have permanent interest in a rumor since he cannot transit to the stifler class automatically, ie. without any contact.

In Huo et al. (2012), the inflow and outflow of individuals to and from the system are both taken into account. Furthermore, unlike models described above, the model considers the human behavior, as we next describe. Upon hearing a rumor, the individual takes time deciding whether the rumor is worth spreading or not. Thus the latter goes either to the "spreader" or "stifler" class respectively. During this phase, the individual is said to be going through the latent period which is termed as the incubation phase. Hereby, investigating the role of incubators in rumor transmission. The authors also prove the existence of the rumor endemic equilibrium. In a recent study due to Horst (2005), it is pointed out that rumor spreading agents change their actions at random points in time at a rate that depends on the current state of some neighborhood and the average situation throughout the entire population.

Because of the existence of such a randomness in the influence that the incubators exert on the class of ignorants, a stochastic model can be more appropriate way of modeling the dynamics of rumor propagation with a given population consisting of ignorant, incubator, spreader and stifler classes. The stochastic version relies on the chance variation in the rate of change of the influence that incubators exert on ignorants. It provides more insight into an individual level modeling and thus takes into consideration a population of reduced size where every individual plays an important role in the model. It is therefore very effective when known heterogeneities are important as in small or isolated populations. The purpose of this paper is to establish and analyze a system of stochastic differential equations describing the dynamics of rumor transmission within a given population.

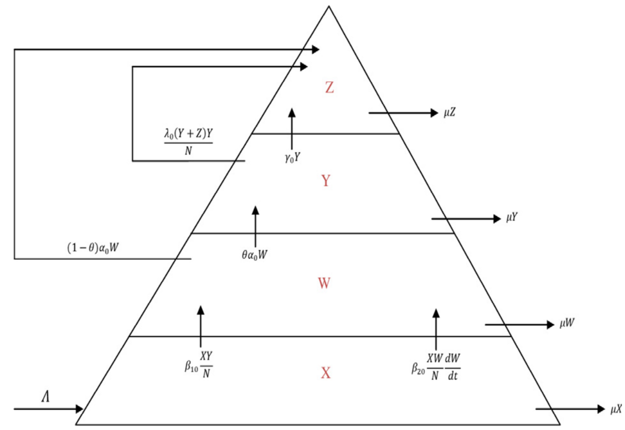

Fig. 1. Schematic representation of a rumor model.

The organization of the paper is as follows. In Section 3, we describe the model for the dynamic rumor transmission with incubators and having a constant immigration. In Section 4, we briefly describe the properties of the deterministic model. The dynamical properties of the stochastic are also studied. In Section 5, we illustrate our results based on numerical simulations. Eventually, we give the concluding remarks.

3. A rumor model with incubation

According to Huo et al. (2012), a given population can be considered as consisting of four categories, namely, ignorant, incubator, spreader and stifler. The proportion of the rumor classes, ie, ignorant, incubator, spreader and stifler are represented as $X(t)$, $W(t)$, $Y(t)$ and $Z(t)$ respectively, such that $X(t) + W(t) + Y(t) + Z(t) = N(t)$, where $N(t)$ denotes the total population size at a particular time, t. Fig. 1 shows a schematic representation of the model. The transition of an individual from a rumor class to another are represented by the different arrows. Fig. 1 gives a realistic point of view since the population of ignorant is much larger than that of spreader. We note that the movement of the individuals from one class to another within the structure is irreversible, i.e. motion is in one direction only. Moreover, we consider the movement into the ignorant class to be a constant $\Lambda > 0$ and that leaving the rumor classes to be $\mu > 0$. The emigration constant μ does not depend on any rumor classes. Upon a successful contact with a spreader or an incubator, an individual leave his "ignorant" class and passes to the "incubator" class, where he distinguishes the integrity of the rumor. Thus, $\frac{\beta_{10}XY}{N}$ and $\frac{\beta_{20}XW}{N}$, respectively represent such contacts. Believing the rumor, the incubator leaves his rumor class to become spreader at a constant rate $\theta \in (0, 1]$. Conversely, he becomes a stifler at a rate of $(1-\theta)$. When two spreaders meet, they transmit a rumor at a constant frequency. On hearing the rumor repeatedly, the spreader gets tired and thus loses interest in it. As a result, he becomes a stifler and we denote this change by $\frac{\lambda_0 Y^2}{N}$. Any contact between a spreader and a stifler outcomes in the spreader