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Short communication

Non-manipulable assignment of individuals to positions revisited[☆]

Tommy Andersson [*], Lars-Gunnar Svensson [1]

Department of Economics, Lund University, Box 7082, SE-222 07, Lund, Sweden

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ABSTRACT

This paper investigates an allocation rule that fairly assigns at most one indivisible object and a monetary compensation to each agent, under the restriction that the monetary compensations do not exceed some exogenously given upper bound. A few properties of this allocation rule are stated and the main result demonstrates that the allocation rule is coalitionally strategy-proof.

1. Introduction

The problem of fairly allocating a number of indivisible objects and some money among a group of agents has received considerable attention in the literature, see, e.g., Alkan et al. (1991), Maskin (1987), Svensson (1983) and Tadenuma and Thomson (1991). One may think of the indivisible objects as jobs, positions, houses, etc. Since the preferences of the agents are private information, the agents

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* Corresponding author. Tel.: +46 (0) 46 222 49 70; fax: +46 (0) 46 222 41 18.
E-mail addresses: tommy.andersson@nek.lu.se (T. Andersson), lars-gunnar.svensson@nek.lu.se (L.-G. Svensson).

1 Tel.: +46 (0) 46 222 86 79; fax: +46 (0) 46 222 41 18.

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may have an incentive to manipulate the allocation rule that is used to solve the assignment problem. This paper shows the existence of an allocation rule that is non-manipulable in the strong sense that it is not possible for any agent or any coalition of agents to manipulate the outcome of the allocation rule by misrepresenting preferences over the objects. Such an allocation rule is said to be coalitionally strategy-proof.

In a recent study, Sun and Yang (2003) investigated a fair allocation problem with an equal number of agents and objects. In their model, each agent must be assigned an object even if it is unprofitable for him, and each object has a maximum compensation limit. In this sense, individual rationality need not be satisfied for the agents, and it is not possible to attach arbitrary monetary compensations to the objects. The main result in Sun and Yang (2003, Theorem 3.1) demonstrates that by regarding the unique fair compensation vector, where the sum of compensations is maximized subject to the maximum compensation limits, as a mechanism for allocating objects among agents, no agents will have an incentive to misrepresent their preferences over the objects, i.e., the outcome is (individually) strategy-proof.

We investigate the model of Sun and Yang (2003) with two important modifications: (i) we allow for any relation between the number of objects and the number of agents and (ii) we cover the cases with and without individual rationality. This is by no means a trivial generalization of the model. We investigate an allocation rule that, in principle, is the same as theirs and our main result (Theorem 1) generalizes and extends the main finding in Sun and Yang (2003, Theorem 3.1), in the sense that we prove that the allocation rule is coalitionally strategy-proof.

Coalitionally strategy-proofness has previously been demonstrated in a multi-object auction model by Demange and Gale (1985, Theorem 2). The potential buyers of the objects, in their multi-object auction model, correspond to the agents in our allocation model, and their Theorem 2, i.e., the analogue of our Theorem 1, states that their mechanism is coalitionally strategy-proof for the buyers. However, our Theorem 1 and their Theorem 2 are not logically identical, because the outcome of their mechanism is always individually rational, whereas we can, but need not, require individually rational outcomes. Hence, our Theorem 1 is an extension to their Theorem 2. In this sense, our result concerning non-manipulability generalizes the related findings in both Sun and Yang (2003, Theorem 3.1) and Demange and Gale (1985, Theorem 2). A last contribution of this paper is of a technical nature, i.e., the proof of the main result is valid for general preferences, and is short and simple.

2. The model and basic definitions

We consider an economy with a finite number of agents and a finite number of objects. The set of agents and objects are denoted by $N = \{1, \ldots, n\}$ and $M = \{1, \ldots, m\}$, respectively. Initially, it is assumed that there are at least as many objects as there are agents, i.e., $\#M \geq \#N$, but this assumption will be relaxed later. There is also a divisible good called money. Each agent $i \in N$ has preferences over consumption bundles $(j, \alpha) \in M \times \mathbb{R}$, represented by a continuous utility function $u_i(j, \alpha)$. The utility function is supposed to be strictly increasing in money. Moreover, for each agent $i \in N$ and for any two bundles (j, α) and (k, α'), there is an amount β of money such that $u_i(j, \alpha) = u_i(k, \alpha' + \beta)$. This means that no object is infinitely good or bad for any agent. A list $u = (u_1, \ldots, u_n)$ of individual utility functions is a (preference) profile. We also adopt the notational convention of writing $u = (u_C, u_{-C})$ for $C \subset N$. The set of profiles with utility functions having the above properties is denoted by $\mathcal{U}$.

An allocation is a list of consumption bundles. It is a pair (a, x), where $a : N \to M$ is an injective mapping assigning object a_i to agent $i \in N$ and where $x \in \mathbb{R}^m$ distributes the quantity x_j of money to object $j \in M$, and, hence, also x_j to agent $i \in N$ if $a_i = j$. We call a the assignment and x the distribution. Each object $j \in M$ has an exogenously given maximum compensation limit, $\bar{x}_j$. These compensation limits are gathered in the vector $\bar{x} \in \mathbb{R}^m$. The set of allocations is denoted by $\mathcal{A}$. An allocation rule is a non-empty correspondence, φ, that, for each profile $u \in \mathcal{U}$, selects a set of allocations, $\varphi(u) \subset \mathcal{A}$, such that $u_i(b_i, y_{b_i}) = u_i(a_i, x_{a_i})$ for all $i \in N$ if $(a, x) \in \varphi(u)$ and $(b, y) \in \varphi(u)$. Hence, the various outcomes in the set $\varphi(u)$ are utility equivalent, and such a correspondence is called essentially single-valued. We end this section with two important definitions.