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An axiomatic characterization of value judgments relative to a reference point

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Abstract

Much theoretical and experimental work suggest that individual preferences depend on the individual's reference point (or status quo, or standard). We present an axiomatic characterization of value judgments relative to a reference point. The derived models present closed formulas for the reference comparison function, and include the reference point. By these functional forms of the possible reference comparison functions, the role of the reference point in the relative judgment is characterized.

Keywords: Reference point; Preference judgment; Functional equations

JEL classification: C-Mathematical and Quantitative Methods; C0-General

1. Introduction

In a *value judgment test situation*, a subject is faced with a set of stimuli, and asked to express his level of preference toward the stimuli in task-specific ways, for example, by indicating on a line how much he likes or dislikes the length of a car, or the amount of money offered for a particular job. We suppose that the sets of all possible stimuli are unidimensional parametric, in the sense that a (not necessarily directly observable) real

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number (e.g., in this example, the length of a car) is attached to them. The real number attached to the stimuli may reflect to a scale of a property of the stimuli (e.g., possible amounts of money offered for a specific job), and will be denoted by x, y, . . ., and called *stimulus parameter*. This underlying property will be called the *domain of the stimuli*, and $x<y$ means that y is more then x in the domain in question.

It is also assumed that in a value judgment test situation, the subject identifies a *reference stimulus* (any stimulus can be chosen as the reference point) and then compares this reference stimulus (with parameter z) with the test stimuli (with parameter x), and the result of the comparison is a real number, denoted by $K(x,z)$. In what follows, K is called a *reference comparison function*. The parameter value z of the reference stimulus plays the role of a kind of *reference point* (or subjective standard; or status quo; or, in utility theory, it is a kind of zero point, i.e., the identity of a binary operation) in which the subject is more or less sure. The result of the comparison $K(x,z)$ is understood as the *subjective value of the reference stimulus over the test stimulus*.

For instance, if someone goes to buy a new car, he may already have an idea about the kind of car he wants, a sort of personal standard against which he compares the quality of a car offered by a sales manager. This potential buyer probably has a greater preference for a car that he considers better than his standard and less preference for a car that is below his standard. In order to articulate the relevance of introducing a reference stimulus and the reference comparison function, see the evidence given by, for example, Tversky and Kahneman (1991), Bateman et al. (1997), Mussweiler (2003a) and Munro and Sugden (2003), among others. A good recent summary can be found in Mussweiler (2003b, pp. 472), where he explicitly states: "In fact, any evaluation is relative in that it refers to a comparison of the evaluated target with a pertinent norm or standard". The comparison function K can be regarded as a model for magnitude estimation (relative to a reference point), and an extensive discussion and recent axiomatization of related models can be found in Falmagne (1985), Narens (1996), and Luce (2002, 2004).

The aim of this paper is to present an axiomatic characterization of unidimensional value judgments relative to a reference point, and if possible to find a closed form of the reference comparison function. In what follows, we characterize a possible model of value judgment by invariance assumptions on the comparison function. In our main theorem (Theorem 2), the resulting functional equations are solved, and the solutions are summarized in Fig. 3. For simplicity, in what follows, a stimulus will be identified with its parameter value.

2. Basic functional equations of comparisons

In order to develop meaningful assumptions about the reference comparison function, we introduce a test situation whereby a subject is asked to compare two stimuli x and y in a specific domain, and to express how much y is more than x. We suppose that this kind of comparison is made in two steps. In the first step, both stimuli x and y are compared to the reference point z by the reference comparison function K, and the results are $K(x,z)$ and $K(y,z)$, respectively. In the second step, the subjective values $K(x,z)$ and $K(y,z)$ are compared by another function H (the domain and the range of H will be specified later in